Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 264 SEPTEMBER, 1957 No. 3

NOTE ON POSSIBLE EFFECT OF ELECTRIC FIELD ON THE GROWTH OF PLANTS

BY

HARLAN T. STETSON[1]

In an article entitled "Cosmic Terrestrial Research," published in the *Scientific Monthly* for March 1944 (Vol. 58, pp. 207–217), four paragraphs were devoted to some preliminary experiments on the growth of plants as modified by an external electric field. Illustrations were given exhibiting what appeared to be an accelerated growth in the case of plants subjected to a negative electric field. So many inquiries have been received seeking further information that it appears desirable at this late date to publish more details of the experiment to supplement the brief account given in the article in the *Scientific Monthly*.

The whole problem involving this excursion into horticulture was instigated by the never answered question as to why vegetation grows perpendicular to the surface of the geoid of the earth. The fact that the direction of growth does not follow a perpendicular to the earth's surface is well exhibited in the growth of trees on a mountain side. The direction of growth in general is that of opposition to the direction of gravity. Careless thinking may at first bring one to the conclusion that the reason for such behavior is the phototropic effect of the sun and sky upon growing things. On second thought, it becomes obvious that a phototropic effect will cause plant life to grow towards the point of maximum illumination, which in middle latitudes is far from the zenith. If the phototropic effect was the principal factor, trees and plants would be deflected from the perpendicular to follow the diurnal path of the sun and maximum sky illumination. Such is not the case. One further illustration that growth takes place against the direction of

[1] Fort Lauderdale, Fla.; formerly, Massachusetts Institute of Technology, Director, Cosmic Terrestrial Research Laboratory, Needham, Mass.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

gravity, independent of any phototropic effect, is the sprouting of bulbs that have been buried in the ground for many months. Every garden lover who resurrects bulbs in early spring is well aware that sprouts, even if initially directed downward, very quickly change their direction of growth to a direction opposite to that of gravity while still buried and, therefore, are not affected by any phototropic factor.

Anyone with even a casual acquaintance with the physics of the earth knows of the existence of an electric field surrounding the earth. The potential of this field varies with the distance upwards from the surface of the earth, attaining in ordinary fair weather a potential of 100–150 volts per vertical meter. Electrical potential between the ground and a given point in the atmosphere not only increases with height but also varies with the time of day, the season of the year, and possibly also with respect to the sunspot cycle. Electricity is almost constantly flowing from the air to the ground with a current strength equivalent to 1000 amperes for the entire surface of the globe. Whether or not the cause of plants growing upward away from the center of the earth is to be attributed to or affected by this electric field no one knows. This was the reason for the subject matter of this article, and such results as were obtained are given for what they are worth.

On the second floor of the Cosmic Terrestrial Research Laboratory at Needham, Massachusetts, a drafting room 17 ft. by 19 ft. 6 in., with a southern exposure, was available for the experiments about to be described.

In planning the initial experiment, careful consideration was given to the fact that both the experimental plant and the control plant should be growing under identical conditions, except that the experimental plant should be subject to an electric field throughout the life of the experiment while in the case of the control plant no electric field was to be imposed.

It was essential that the illumination of both specimens should be as nearly the same as practicable. A large plate glass window near which the specimens were to be grown seemed to provide the answer as to equivalent illumination to both the control and the experimental specimens. To insure that the food and water supply should be the same at all times, it was decided to grow the plants hydroponically. Both specimens were fed automatically from an inverted flask arrangement. A nutrient solution dispensed by the flasks is described in a paper entitled "The Water-Culture Method for Growing Plants Without Soil," by D. R. Hoagland and D. I. Arnon of the California Agricultural Experiment Station. The paper is published in the annual report of the Smithsonian Institution for 1938, pp. 461–487. The solution used in the present experiments was that of Solution 2, described on pp. 485–487. Distilled water and pure chemicals were deemed important.

There is a variety of hydroponic solutions that would probably

prove as effective as that here used. One may now buy a combination of chemicals for water-culture that would probably prove equally satisfactory when the proper amount of water is added. The procedure of growing the specimen plants hydroponically eliminates differences in soil in the planting box and irregularity in hand watering. Many papers have been written with respect to soil-less growth of plants and procedures outlined in detail. Those interested should refer to the literature in the field.

For receptacles for the plants under study, small shallow cans with a lining of melted paraffin were provided. The purpose of applying the paraffin was to avoid any chemical reaction between the nutrient solution and the container. Connections between these prepared receptacles and the reservoirs fed from the inverted flasks containing the nutrient solution were accomplished through short lengths of glass tubing with rubber gaskets at each end. Each "can" receptacle was two-thirds filled with reasonably sterile gravel to provide anchorage for growing roots. A thin layer of absorbent cotton topped the gravel in each receptacle and provided a suitable moist bed for the seedlings to sprout.

In order to create an electric field, a square of copper screening approximately 15 cm. by 15 cm. was supported 20 cm. above the rim of the can receptacle by a wooden dowel rod. The two receptacles for convenience were labeled A and B. "A" provided for the experimental plant while "B," a short distance from "A," provided for the control specimen. It is to be noted that a copper screen was erected over the control as well as over the experimental specimen, but no voltage was applied on the screen of the control. In the case of the experimental specimen a conventional "B" battery producing 45 volts maintained an electrical potential between screen "A" and the can "A." Connecting wires were soldered to the copper screen and connection made to the negative pole of the "B" battery. The positive pole was similarly connected to the can receptacle containing the nutrient solution. Thus is maintained an electric field between the roots of the plant "A" and the copper screen 20 cm. above the receptacle. There was no electrical connection in the case of the control specimen.

The first actual planting occurred on April 8, 1941, at 3 p.m., when marigold tomato seeds were planted in each receptacle "A" and "B." The seeds were dropped on the surface of the wet absorbent cotton and after planting, the receptacles were overlaid with a square of blue blotting paper to provide shadow from the bright sky until sprouting occurred. On April 17, five of the seven seeds planted in "A" had sprouted. Six seeds in the control receptacle "B" were also found sprouted. The voltage on "A" was now increased from a -45 volts to -90 volts on the screen with respect to the can receptacle.

Daily inspections of the experiment were made, and by April 27,

1941 it was noted that the experimental seelings in can "A" were outstripping the growth in the control can "B." In order to obtain some rough quantitative estimate of growth, a mirror reading device was installed for estimating height of growth above the base of the receptacles. On the April 27 observations, the following readings were made: In receptacle "A" subject to the electric field, the two tallest shoots read 9.3 cm. above shelf and 8.6 cm. above shelf. In the control receptacle "B," the two highest shoots read 7.0 cm. and 6.8 cm.

On May 29 an entry in the original notebook reads, "experimental plant has grown luxuriously; control poorly developed." The experimental plant was removed and placed outside the laboratory building. The control specimen was scrapped.

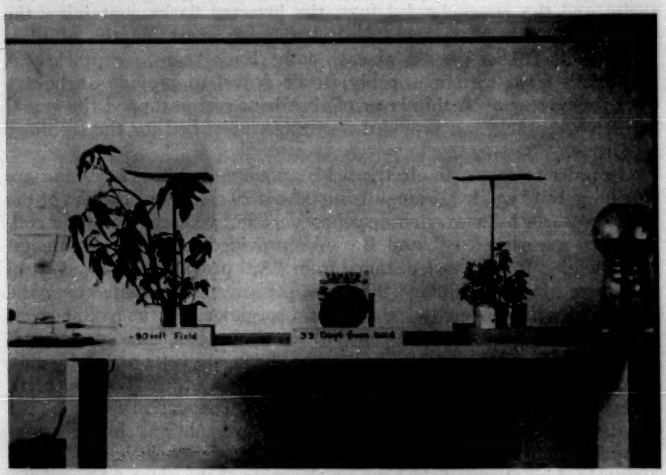


FIG. 1. Tomato, 33 days from seed.

Whatever effect, if any, the electrical field had in promoting the observed growth, the observer was concerned with the possibility that the two plants, in spite of precautions, had been affected by unequal illumination. It seemed possible that the more western container received slightly stronger illumination from the sun and sky than the eastern receptacle. Accordingly, on May 29, 1941 a second planting was made in every way the same as the procedure previously outlined except that the locations of "A" and "B" were interchanged. On June 10, twelve days after planting, it was found that again the plant showing the greater growth was the one subject to the electric field imposed. Thus any effect unequal illumination may have had, was more than

offset by some other factor, presumably the electric field. Both plants showed a strong phototropic effect.

On July 2, thirty-three days after planting, a photograph was taken (Fig. 1) showing the greater growth of the experimental specimen, subject to the electric field, as compared with the control specimen without an electric field. As these plantings had outgrown the apparatus, the specimens were removed from the laboratory.

On September 8, 1941, another planting was made. This time mustard seeds were used in place of tomato seeds on account of shorter period of growth to maturity. Before planting, the apparatus was thoroughly cleaned and reconditioned. The receptacles were filled to

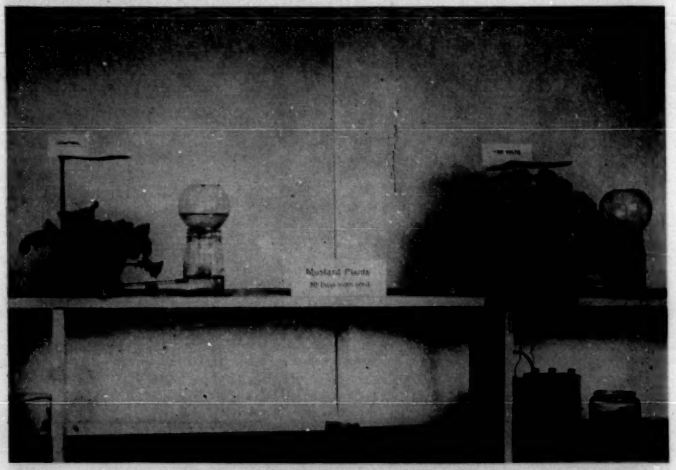


FIG. 2. Mustard plants, 38 days from seed.

the usual level with washed and baked sand. This time the east receptacle was charged with 90 negative volts. The west receptacle acting as a control was without a potential gradient. The same nutrient solution was used in maintaining the same level of fluid in the receptacles. Squares of blotting paper were again used as covers for the receptacles during the early germination.

On September 11, the seeds had sprouted and the blotting paper covers were removed.

On October 16, thirty-eight days from seed, the growth of the experimental specimen had far exceeded that of the control. To exhibit this difference in growth, a photograph was taken, see Fig. 2. The growth

was allowed to continue until October 20, when the plants had outgrown the containers.

In order to remove any further doubt as to the effect of nonuniform illumination, a mechanical device was constructed with four radial arms, each of which was capable of supporting a receptacle fed with the nutrient solution from inverted flasks (Fig. 3). The whole affair was sym-

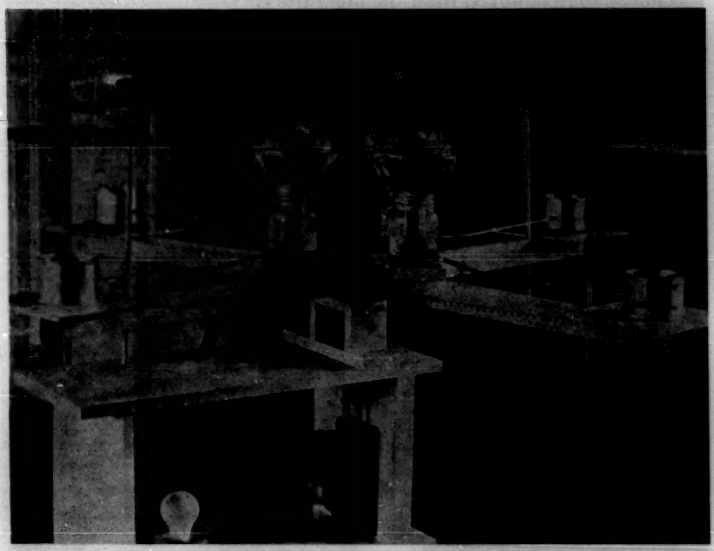


FIG. 3. Rotogrower device.

metrical with respect to a verticle axis about which the four arms were continuously rotated by a telechron motor. It was believed that with continual rotation any unequal illumination would be shared equally by all four specimens each revolution. The four receptacles on the four arms of this special "revolving grower" were prepared as previously. Tomato seeds were planted in each of the four receptacles. The voltages applied to the screen above the receptacles were as follows: number one, −40 volts; number two, −75 volts; number three, −190 volts; number four, control, no voltage. As before, all the screens were supported 20 cm. above the gravel level in the receptacles.

The plantings were made on October 24, with each receptacle containing seven seeds. On November 22, photographs were taken of each individual receptacle showing the state of growth thirty days after planting (Figs. 4, 5, 6, 7). Inspection of the photographs shows that

FIG. 4. Tomato, 30-days growth, −40 volts.

maximum growth occurred where the electric field's gradient was −40 volts between the screen and the can. The next most favored growth corresponds to −75 volts gradient. Receptacle number three, with the

FIG. 5. Tomato, 30-days growth, −75 volts.

FIG. 6. Tomato, 30-days growth, −190 volts.

maximum gradient of −190 volts, showed greatly retarded growth; in fact, the seedlings appeared little more than sprouted. The result of this experiment, providing the same integrated illumination for each

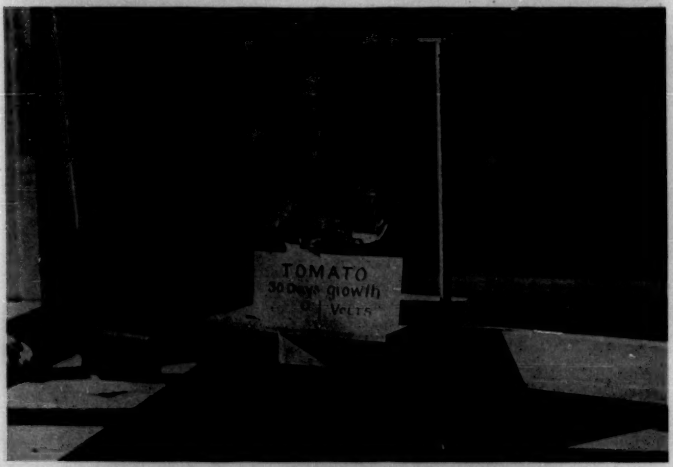


FIG. 7. Tomato, 30-days growth, 0 volts (control).

plant, appears to have eliminated any question of unequal phototropic effects, had they been present.

It is to be noted that the experiments reported thus far involve arbitrary voltages chosen by convenience. In order to determine the optimum voltage for growth, if such exists for any given planting, it was decided to construct a special planting box to which a voltage could be applied that would impose a variable field gradient throughout the duration of the experiment. Accordingly, on Dec. 8, 1941 a box was constructed approximately 1 m. long and 10 cm. wide, above which a strip of copper screening was erected forming the hypotenuse of a right angle triangle, the base of which was formed by the wooden planting box. The wood was saturated with paraffin in order that the container should be water-repellent. Near the bottom of the box, a brass rod extended the entire length of the box, projecting at one end sufficiently to allow soldering a connecting wire leading to the positive pole of the battery. The negative pole of the battery provided a potential of −190 volts to the screen. The slanting copper screen was 5 cm. above the level of the planting bed of the box at one end and 50 cm. above seed level at the other end of the box. The box was filled with the usual washed gravel on which mustard seeds were planted at intervals of 5 cm. throughout the length of the box. The plants were fed hydroponically with the same nutrient solution as was used before. The strength of the field at the low end of the screen was approximately 38 volts per vertical centimeter and 3.8 volts per vertical centimeter at the upper end. It was early noted that the mustard plants after sprouting showed greater growth near the middle of the box. Growth was allowed to continue and photographs were made to register the curve of growth against the variable field (see Figs. 8, 9). It will be observed in the photographs that the plants at the end of the box subject to a low electric field exhibited near normal growth, whereas at the other end of the box subject to a very strong field the seedlings had developed little above the sprouting stage. The optimum growth occurred where the gradient was approximately 4 volts per centimeter. It is interesting to note that in the early experiments where the voltages were chosen at random, the field was approximately 80 volts for the 20 cm. between the root level and the electrically charged screen overhead. This is the equivalent of 4 volts per centimeter which checks very nicely with the optimum voltage derived from the experimental growth under the slanting screen.

The results from the laboratory plantings reported thus far would appear to indicate that similar experiments conducted in the open would produce comparable results.

In the summer of 1942, through the cooperative interest of an acquaintance, tomato plants were grown in an especially designated area on a farm in Manchester, Vermont, some of the specimens being subjected to the artificially imposed electric field and other plants without

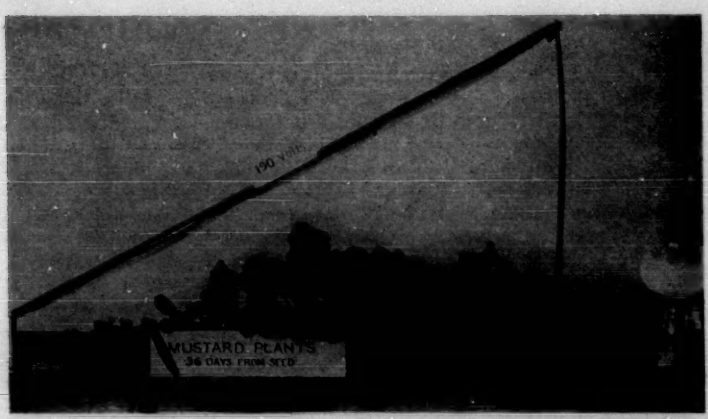


FIG. 8. Mustard plants, 36 days from seed, −190 volts.

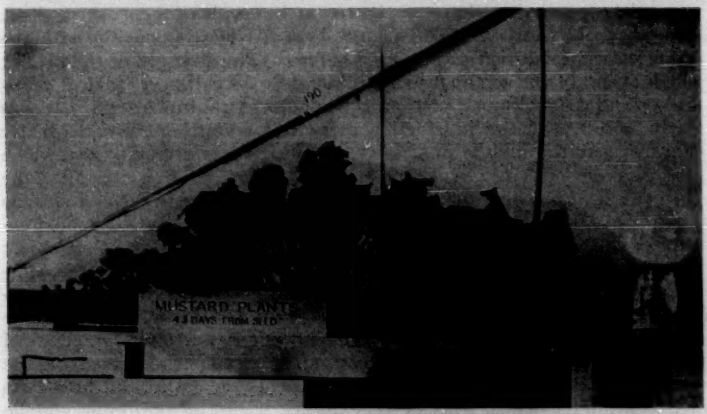


FIG. 9. Mustard plants, 43 days from seed, −190 volts.

such device used as a control. After a month's growth, no significant differences were noted between the plants grown with an electric field and those grown without. Later the author learned on inquiry that tomato seeds were not planted in the ground, but rather, young plants were set out so that the original germinations were not subject to the same experimental condition as those used in the laboratory.

In the summer of 1944, through the interest of Mr. G. F. Prideaux of the General Electric Company, an experiment was carefully supervised at Nela Park, Cleveland, Ohio. In this case, a slanting screen was erected to which a −180 volts was applied to simulate the conditions depicted in Figs. 8 and 9. After a sufficient lapse of time, photographs were made of both the experimental plants and the controls and prints were forwarded to the author of this article for comment. No significant effect could be detected. These negative results in the open present a puzzling problem to account for the differences in the results. It has been suggested that where there is a free circulation of air, as is the case of experiments made out of doors, the ionic gradient is disturbed thereby. This is not the case in a draftless room in the laboratory where the difference in growth was so obvious.

Another point of difference lies in the fact that the electric field indoors was maintained between a root container and the overhead screen without any attempt to maintain the roots at an electrical "ground" level. This suggested a variation in the experiment under conditions where the root containers in the laboratory are connected to the earth through an electrical ground wire.

This was done in the fall of 1942, the results of which are shown in Fig. 10. Referring to Fig. 10, the plant at the extreme left was sub-

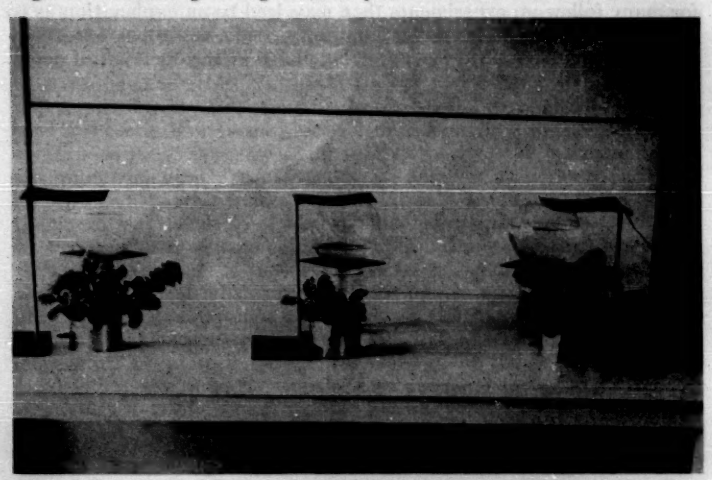


FIG. 10. Three-specimen experiment with ground wire. Photograph of October 7, 1942, mustard, 28 days from seed.

A—extreme right-hand plant, −75 volts, no ground
B—middle plant, no field, no ground
C—extreme left-hand plant, −75 volts, plus terminal (receptacle) grounded

jected to a potential difference between the root receptacle and the overhead screen of −75 volts. The positive terminal of the battery supplying the −75 volts was connected not only to the metallic receptacle in which the seeds were planted, but also to a ground wire leading to the earth. The plant at the extreme right was subjected to a negative potential of −75 volts at the screen with respect to the root receptacle, but without any ground wire. The plant in the middle position in the photograph had neither ground nor battery connections, and was used solely as a control. The photograph was made 28 days after planting. It appears that both in the extreme left and extreme right specimens growth was promoted by the electrical field as compared to the middle control specimen having no imposed field. The plant at the extreme right subject to −75 volts but without ground wire showed growth comparable with the plant at the extreme left subject to −75 volts with the root receptacle grounded. So far as this experiment is concerned, the electrical potential of the roots with respect to the earth would appear to be unimportant.

Other experiments were planned, but on account of the demand for space for other investigations under the pressure of the national emergency the experiments on plant growth were discontinued. It is hoped that the detailed account here presented may stimulate sufficient interest for many follow-up experiments that may lead to an explanation as to why the behavior pattern under laboratory conditions appears so different from results obtained from growing plants in the open out of doors.

ORBITAL BEHAVIOR OF EARTH SATELLITES

BY

ROBERT E. ROBERSON [1]

Part I *

ABSTRACT

Although the methods of classical astronomy are applicable to the analysis of orbits of close satellites, a need has existed for a treatment of intermediate accuracy of the effect of the oblateness of the earth, a major perturbation source under these conditions. This paper reviews some of the more recent contributions toward this goal, compiles a number of orbital results of value in satellite engineering, and discusses some of the perturbation sources on a satellite orbit. It develops a solution of first order in the oblateness for orbits about an oblate spheroid by a method similar to Hansen's classical perturbation method. Particular attention is given to the effect of initial conditions on the orbit elements.

1. INTRODUCTION

Current interest in earth satellites has been accompanied by an increasing literature on the major problems of satellite engineering. Much of this literature is descriptive or semi-quantitative, but some problems also have received considerable analytical attention. One of the latter is the problem of the satellite's orbital motion.

An adequate orbit analysis for earth satellites is of fundamental importance for several reasons. It is essential if the satellite is to be used as a geodetic or geographic tool, or as a research vehicle for which a continuous position record is an integral part of the instrumentation. By minimizing search requirements, the analysis is the key to successful location and tracking from a ground station. It is a central question in the operational analysis of almost all satellite applications.

For minimum instrumented satellites and some research satellites a very simple picture of the orbit suffices—a circular or elliptical path about a spherical earth, all perturbations being ignored. However, most applications require a somewhat more sophisticated picture. The question of life-time on orbit usually is of interest, so that air drag considerations must be added. The effect of the earth's oblateness often is important, such as in the case described in (6)[2] and (1) where this effect is used to improve our knowlecge of the ellipticity of the earth. Still other perturbations of the orbit, such as the effect of tidal friction (7), have been claimed (incorrectly) to be of operational importance.

[1] Autonetics Division, North American Aviation, Inc., Bellflower, Calif.

* Part II will be published in this JOURNAL for October, 1957.

[2] The boldface numbers in parentheses refer to the references appended to Part I.

The present status of satellite orbit analysis can be summarized as follows.

1. The classical astronomical literature (for example (3)) does not adequately describe the orbit of a close satellite. It concerns primarily satellites which are remote enough from the primary body that effects like air drag and oblateness are overshadowed by the perturbations of the sun and major planets. In cases where oblateness is considered, results generally are restricted to orbits with low inclination. Furthermore, there are other characteristics of these analyses which make it difficult to apply them to the motion of artificial satellites.

2. The major problems of air drag analysis are what drag coefficient and air density function to use, and the purely formal difficulty of solving the nonlinear differential equations of motion. A recent paper (10) discusses these questions in some detail, but only solves the equations of motion for a case of spiral orbits which is not of major practical importance.

3. As mentioned above, there have been some analyses encompassing the effect of oblateness of natural satellites, but unfortunately these are not entirely satisfactory for present purposes. A treatment by Spitzer (12) with the artificial satellites in mind considers only the perturbations of a circular orbit. De Nike (4) discusses computer solutions for the oblateness effect and for an orbit under the combined action of oblateness and air drag. Another recent paper (1) discusses the use of the oblateness perturbation for geodetic purposes, but is seriously deficient from several points of view. Krause (6), in a paper which does not seem to be as well known as it deserves to be, displays the equations of perturbation of a set of orbital elements and discusses the oblateness effect among other topics.

4. Solutions to the two-body problem are abundant, for in addition to their sufficiency for many applications, and consequent intrinsic interest, they usually serve as generating solutions in the analysis of small perturbations. However, there seems to be no compendium of results for this simple case which includes all principal expressions of value in satellite engineering and the interrelationships among the many available parameter sets in terms of which these expressions are commonly written.

5. Finally, I am not aware of any general discussion and assessment of the miscellaneous small perturbation forces which can be conceived to act on artificial satellites; none, at least, in company with a quantitative description of the orbital behavior under the major perturbations. However, some treatments of particular perturbations have been given, such as those of Krause (6) and Spitzer (12) on the effects of the sun and moon.

As implied by this status summary, a need exists for an integrated and analytical treatment of the orbit of an earth satellite with the re-

quirements of satellite engineering rather than classical astronomy primarily in mind. The present paper is directed toward that need, attempting to provide a unified treatment of the major results which presently are scattered rather widely throughout the astronautical literature.

The preliminary considerations below include the definition of a reference coordinate system, discussions of the sources of orbital perturbations and the rationale under which the orbit is analyzed, and estimates of the effects of the minor perturbations. Section 2 also summarizes the principal results for elliptic motion and gives tabular relations among the various sets of parameters in terms of which it is convenient to describe elliptic motion.

The bulk of the paper is concerned with an analysis of one of the two major perturbations of a close satellite—that of the earth's oblateness.[3] Results are of intermediate accuracy, which is to say they are correct to first order terms in the ellipticity of the earth. Thus they suffice for most satellite engineering purposes and for operational studies, although they may not be adequate for the most precise astronomical needs, such as that of very long time prediction of position.[4] In particular, all important aspects of the motion arising from the oblateness of the earth are encompassed. I feel that higher order approximations would serve principally to make slight adjustments in some of the parameter values such as orbital frequency.

Air drag, the other major perturbation source, has been analyzed in comparable detail. However, the combined length of the two analyses and the fact that the treatment of air drag is an inherently self-contained topic have made it preferable to present the latter as a separate, companion paper.

2. PRELIMINARY CONSIDERATIONS

2.1 Coordinate Systems

A variety of coordinate systems is used in the astronomical literature to describe satellite position. (See, for example (3).) Although these coordinate systems sometimes appear numerous to the point of confusion, they reflect the wealth of analytical methods which have been developed for the approximate solution of the dynamical equations of motion, invariably nonlinear in celestial mechanics. There tends to be one set of generalized coordinates which lends itself best to each such approximate method.

It has been observed previously that one way in which artificial satellite orbit analysis differs from classical orbit theory is that orbits having very high inclinations, such as meridian orbits, must be treated as commonplace. Therefore, rather than using the equator as a basic

[3] It should be mentioned that this part of the analysis has been distributed as a technical report after an original publication in that form in January, 1953.

[4] A forthcoming book by Dr. Samuel Herrick is likely to provide a definite treatment of this problem from the astronomer's viewpoint.

reference plane, as is done in a number of treatments of satellite orbits, I have used a plane inclined to the equator at an angle γ which is not assumed to be small. Furthermore, because it is well known that the oblateness of a body causes the "plane" of its motion to rotate about the body's axis of symmetry, the inclined plane mentioned above is assumed to rotate about the earth's polar axis with an unspecified angular velocity. The rationale of choosing this angular velocity is discussed in Section 2.3 below.

The coordinate system introduced below is well suited to the particular perturbation method used later to analyze the major aspects of the orbit behavior. At the same time, it is one which seems to have intuitive appeal to persons with engineering backgrounds.

Let $\mathfrak{X}\mathfrak{Y}\mathfrak{Z}$ be a right-hand rectangular Cartesian coordinate system with its origin at the center of the earth and an orientation fixed in inertial space. Let the $\mathfrak{Y}$ axis lie in the direction of the earth's north pole. Although the orientation of $\mathfrak{X}$ and $\mathfrak{Z}$ is arbitrary, assume for definiteness that the $\mathfrak{Z}$ axis is directed toward the First Point of Aries.[5] Next, let $X\ Y\ Z$ be a similar system obtained from $\mathfrak{X}\ \mathfrak{Y}\ \mathfrak{Z}$ by a rotation through an angle Λ_0 about the $\mathfrak{Y}$ axis. The significance of the notation chosen for this angle becomes apparent later. These two coordinate systems, the relation between them, and their relation to the earth are shown in Fig. 1.

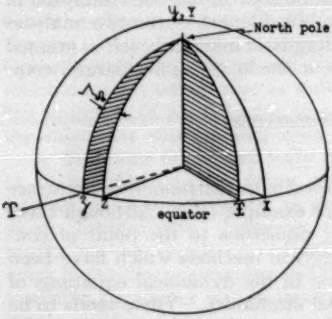


FIG. 1. Coordinate systems $\mathfrak{X},\mathfrak{Y},\mathfrak{Z}$ and XYZ in relation to the earth.

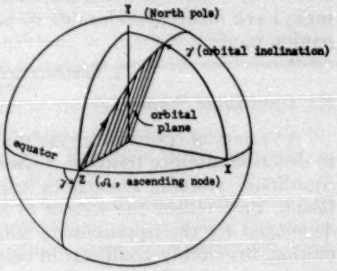


FIG. 2. Orbital plane and its relationship to X, Y, Z.

Next consider a plane containing the Z axis and inclined to the earth's equator at an angle γ (with $0 \leq \gamma \leq \pi$), as shown in Fig. 2. An important problem of orbit analysis is to find how the angle Λ_0 must behave if the satellite is to remain within, or close to, this inclined plane.

[5] For the non-astronomer, the First Point of Aries, symbolized by ♈, is the intersection on the celestial sphere of the earth's equator and the sun's path as it passes from south to north across the equator, that is, the vernal equinox.

Thus it is natural to speak of the plane as the "orbital plane" and of the angle γ as the "orbital inclination." The convention is adopted that the satellite crosses the equator from south to north near the Z axis, so

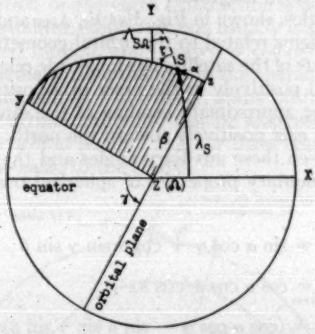


FIG. 3. Position angles of the satellite.

that the intersection of this axis with the projection of the equator on the celestial sphere is the so-called "ascending node"[6] of the orbit, and the Z axis itself is the "line of nodes."

Three generalized coordinates are required to specify completely the location of the satellite relative to the earth. One of them is chosen to be the radial distance, r, of the center of mass of the satellite from the center of the earth. The other two are angular coordinates, α and β, which are described conveniently in terms of the orbital plane.

Figure 3, an edge-on view of the orbital plane from the positive Z axis, shows the satellite at point S. Denote by y the axis through the center of the earth normal to the orbital plane, its positive sense on the northerly side of this plane for $\gamma < \pi/2$ and on the southerly side for $\gamma > \pi/2$. The shaded plane in Fig. 3 contains both the y axis and the satellite and intersects the orbital plane in a line defined as the z axis. The positive sense of this axis is such that it makes an angle of less than $\pi/2$ with the ray from the center of the earth to the satellite. In these terms the dihedral angle from the Zy plane to the yx plane, labeled β, describes the angular advance of the satellite from its ascending node. The angle α, measured in the yz plane positively as shown in the figure, describes the satellite departure from the orbital plane.

Angles α and β clearly bear a relation to the orbital plane similar to that borne by conventional latitude and longitude relative to the equator. The ascending node plays the same role in establishing a zero

[6] This ascending node is customarily designated by Ω in astronomical literature, whence the motivation for the longitude angle notation $\Delta\Omega$. Note that Υ is the ascending node of the sun in its apparent motion about the earth.

of "longitude" in the inclined plane as the Greenwich meridian in establishing longitude on the earth. However, the range of β is supposed to be $-\infty < \beta < \infty$ rather than restricted between 180°E and 180°W as is the case with earth longitude.

Three other angles, shown in Fig. 3 as λ_S, $\Lambda_{S\Omega}$, and ζ, are of frequent occurrence in problems related to the orbital geometry. Respectively, these are the latitude of the satellite; its longitude relative to the ascending node, measured positively in the sense of a positive rotation about the Y axis; and the approximate heading of the satellite's trajectory,[7] measured from the east positively toward the north.

Relations between these auxiliary angles and the basic set are easy to derive from elementary properties of spherical triangles. For reference, they are

$$\sin \lambda_{S\Omega} = \sin \alpha \cos \gamma + \cos \alpha \sin \gamma \sin \beta \tag{1}$$

$$\cos \Lambda_{S\Omega} = \cos \alpha \cos \beta / \cos \lambda_S \tag{2}$$

$$\cos \zeta = (\cos \alpha \cos \gamma - \sin \alpha \sin \gamma \sin \beta) / \cos \lambda_S \tag{3}$$

It is useful to have explicit direction cosine matrices between the coordinate sets $\mathfrak{X}\ \mathfrak{Y}\ \mathfrak{z}$, $X\ Y\ Z$ and $x\ y\ z$. (In the last, x is adjoined to the y and z defined previously so as to form a right-hand system.) These matrices are shown in Table I.

TABLE I. *Direction Cosine Matrices for Coordinate Sets* $\mathfrak{X}\mathfrak{Y}\mathfrak{z}$, XYZ, *and* xyz.

	X	Y	Z	z	y	x
$\mathfrak{X}$	$\cos\Lambda_\Omega$	0	$\sin\Lambda_\Omega$	$-\sin\beta\sin\Lambda_\Omega+\cos\beta\cos\Lambda_\Omega\cos\gamma$	$-\sin\gamma\cos\Lambda_\Omega$	$\cos\beta\sin\Lambda_\Omega+\sin\beta\cos\Lambda_\Omega\cos\gamma$
$\mathfrak{Y}$	0	1	0	$\sin\gamma\cos\beta$	$\cos\gamma$	$\sin\gamma\sin\beta$
$\mathfrak{z}$	$-\sin\Lambda_\Omega$	0	$\cos\Lambda_\Omega$	$-\sin\beta\cos\Lambda_\Omega+\cos\beta\sin\Lambda_\Omega\cos\gamma$	$\sin\gamma\sin\Lambda_\Omega$	$\cos\beta\cos\Lambda_\Omega-\sin\beta\sin\Lambda_\Omega\cos\gamma$
X	1	0	0	$\cos\gamma\cos\beta$	$-\sin\gamma$	$\cos\gamma\sin\beta$
Y	0	1	0	$\sin\gamma\cos\beta$	$\cos\gamma$	$\sin\gamma\sin\beta$
Z	0	0	1	$-\sin\beta$	0	$\cos\beta$

2.2 Elliptic Motion

Consider the orbit that would exist if the earth were a gravitating sphere and exerted the only force on the satellite. The orbital plane would be the plane containing the satellite and the center of the earth at any instant, so oriented that it would also contain the satellite velocity vector at that instant. The plane would remain fixed in inertial space (Λ_Ω and γ both being constant). Moreover, the satellite would follow an elliptic path with the earth at one focus, and would never deviate from the orbital plane.

[7] This interpretation of ζ can be justified later.

Actually, a number of physical effects perturb the orbit from this basic elliptic form. It is the analysis of the more significant perturbations which is the central problem of this paper. However, because the inverse square portion of the earth's attraction is ordinarily the dominant force on the satellite, elliptic motion usually is a good first approximation to the actual motion, and it often is convenient to discuss the actual motion in terms of slow changes in the characteristic parameters of the approximating ellipse. Moreover, many problems related to the applications of an artificial satellite can be analyzed as if the motion were elliptic in the orbital plane. It is for these reasons that the topic of elliptic motion is basic to the study.

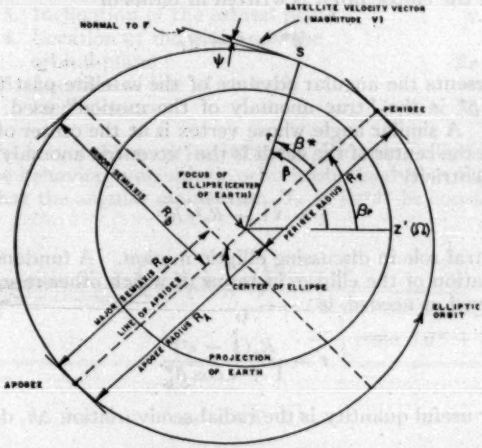


FIG. 4. An elliptical orbit.

It is not my purpose to develop all of the well-known properties of particle motion in an inverse square field—these appear in detail in almost every elementary mechanics and astronomy text. However, there is a definite advantage in at this point giving a few essential definitions, introducing some additional notation, and summarizing certain basic formulas in a form most convenient for the engineering of artificial satellites.

Figure 4 shows the geometry and important parameters of an elliptic orbit. It depicts the face of the orbital plane on which β advances counterclockwise, so that it is a normal view of the northerly side of this plane as it is drawn in Fig. 3. The line of nodes again is shown as the Z axis and the satellite position as S. Because this is a case of plane

motion, the angle α is identically zero and the satellite is located uniquely by the radius r and the angle β. The point of closest approach of the orbit to the center of the earth is called the perigee. The point of furthest departure from the earth, lying diametrically opposite the perigee, is called the apogee. The line connecting perigee and apogee is called the line of apsides.

The size and shape of the ellipse are completely characterized by the lengths R_a and R_b of the major and minor semiaxes, respectively, or equally well by the perigee and apogee distances, R_P and R_A, respectively. Its orientation in the orbital plane can be described conveniently by the angle β_P, from the Z axis to the perigee radius. The equation of the ellipse often is written in terms of

$$\beta^* = \beta - \beta_P \tag{4}$$

which represents the angular advance of the satellite past the perigee. The angle β^* is the "true anomaly of the motion" used in celestial mechanics. A similar angle whose vertex is at the center of the ellipse rather than the center of the earth is the "eccentric anomaly."

The eccentricity

$$\epsilon = \sqrt{1 - R_a^2/R_b^2} \tag{5}$$

plays a central role in discussing elliptic motion. A fundamental form for the equation of the ellipse, in terms of which other representations can be derived as needed, is

$$r = \frac{R_a(1 - \epsilon^2)}{1 + \epsilon \cos \beta^*}. \tag{6}$$

Another useful quantity is the radial semivariation Δh, defined as

$$\Delta h = \frac{1}{2}(R_A - R_P). \tag{7}$$

It is related to ϵ by $\Delta h = \epsilon R_a$. In view of these relations (Eqs. 5, 7 and those implied by the geometry of Fig. 4), it is possible to describe the orbit by any two of the six parameters R_a, R_b, R_A, R_P, ϵ and Δh. However, it also is frequently useful to know how these geometric parameters are related to the dynamic or kinematic parameters of the orbit. The latter include apogee velocity V_A, perigee velocity V_P, angular momentum H_S, and total energy E_S. In particular, the latter two are widely used in elementary mechanics texts.

The orbit can be characterized by any two of the four parameters of the latter set, as well as any two of the previous six. Note, though, that one cannot freely use any pair from the totality of ten parameters.

For example, because $E_S = -K/2\,R_a$, the pair (R_a, E_S) is equivalent to but one parameter.

Table II provides conversions between any selected pair of geometric parameters and the remaining four geometric parameters. Table III gives, for each of these same pairs, values of the four dynamic parameters. It is not especially useful to list the dynamic parameters as the independent variables or to extend the tables to the admissible mixed pairs of geometric and dynamic parameters.

In this paper the following set of orbital elements is chosen as basic:

1. Major semiaxis	R_a
2. Eccentricity	ϵ
3. Inclination of the orbital plane	γ
4. Location of the perigee in the orbital plane	β_P
5. Location of line of nodes in inertial space	Λ_Ω

Only the first two of these are important in discussing elliptic motion.

The time behavior in an elliptic orbit is obtained from Eq. 6 and the condition that the angular momentum $H_S = mr^2\dot{\beta}^*$ be constant.[8] The result is

$$\omega_0 t = \beta^* - 2\,\epsilon \sin \beta^* + \left(\frac{3}{4}\,\epsilon^2 + \frac{1}{8}\,\epsilon^4\right) \sin 2\,\beta^* - \frac{1}{3}\,\epsilon^3 \sin 3\,\beta^* + 0\,(\epsilon^4), \quad (8)$$

where

$$\omega_0 = \sqrt{K/R_a^3} \quad (9)$$

is the so-called "orbital frequency." Reversing the series,

$$\beta^* = \omega_0 t + \left(2\,\epsilon - \frac{1}{4}\,\epsilon^3\right) \sin \omega_0 t + \left(\frac{5}{4}\,\epsilon^2 - \frac{11}{24}\,\epsilon^4\right) \sin 2\,\omega_0 t + \frac{13}{12}\,\epsilon^3 \sin 3\,\omega_0 t + 0\,(\epsilon^4). \quad (10)$$

The orbital radius from Eq. 6 can be rewritten as

$$\frac{r(\beta^*)}{R_a} = \sqrt{1-\epsilon^2}\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{-\epsilon}{1+\sqrt{1-\epsilon^2}}\right)^n \cos n\beta^*\right] \quad (11)$$

[8] The initial instant of time is chosen to be the instant when $\beta^* = 0$.

TABLE II.—*Conversion Between Parameters of Elliptic Orbit.*

Assigned Parameter Pairs	Desired Parameters: Major Semiaxis R_a	Minor Semiaxis R_b	Apogee Radius R_A	Perigee Radius R_P	Eccentricity ϵ	Radial Semivariation Δh
R_a, R_b			$R_a + \sqrt{R_a^2 - R_b^2}$	$R_a - \sqrt{R_a^2 - R_b^2}$	$\sqrt{1 - \frac{R_b^2}{R_a^2}}$	$\sqrt{R_a^2 - R_b^2}$
R_a, R_A		$\sqrt{2R_AR_a - R_A^2}$		$2R_a - R_A$	$\frac{R_A - R_a}{R_a}$	$R_A - R_a$
R_a, R_P		$\sqrt{2R_PR_A - R_P^2}$	$2R_a - R_P$		$\frac{R_a - R_P}{R_a}$	$R_a - R_P$
R_a, ϵ		$R_a\sqrt{1 - \epsilon^2}$	$R_a(1 + \epsilon)$	$R_a(1 - \epsilon)$		ϵR_a
R_a, Δh		$\sqrt{R_a^2 - \Delta h^2}$	$R_a + \Delta h$	$R_a - \Delta h$	$\frac{\Delta h}{R_a}$	
R_b, R_A	$\frac{1}{2}\left(\frac{R_A^2 + R_b^2}{R_A}\right)$			$\frac{R_b^2}{R_A}$	$\frac{R_A^2 - R_b^2}{R_A^2 + R_b^2}$	$\frac{R_A^2 - R_b^2}{2R_A}$
R_b, R_P	$\frac{1}{2}\left(\frac{R_P^2 + R_b^2}{R_P}\right)$		$\frac{R_b^2}{R_P}$		$\frac{R_b^2 - R_P^2}{R_b^2 + R_P^2}$	$\frac{R_b^2 - R_P^2}{2R_P}$
R_b, ϵ	$\frac{R_b}{\sqrt{1 - \epsilon^2}}$		$R_b\sqrt{\frac{1 + \epsilon}{1 - \epsilon}}$	$R_b\sqrt{\frac{1 - \epsilon}{1 + \epsilon}}$		$\frac{R_b\epsilon}{\sqrt{1 - \epsilon^2}}$

TABLE II.—*Continued*

Assigned Parameter Pairs	Desired Parameters: Major Semiaxis R_a	Minor Semiaxis R_b	Apogee Radius R_A	Perigee Radius R_P	Eccentricity ϵ	Radial Semi-variation Δh
R_b, Δh	$\sqrt{R^2_b + \Delta h^2}$		$\sqrt{R^2_b + \Delta h^2} + \Delta h$	$\sqrt{R^2_b + \Delta h^2} - \Delta h$	$\sqrt{1 - \frac{R^2_b}{R^2_b + \Delta h^2}}$	
R_A, R_P	$\frac{1}{2}(R_A + R_P)$	$R_A R_P$			$\frac{R_A - R_P}{R_A + R_P}$	$\frac{R_A - R_P}{2}$
R_A, ϵ	$\frac{R_A}{1 + \epsilon}$	$R_A\sqrt{\frac{1 - \epsilon}{1 + \epsilon}}$		$\frac{R_A(1 - \epsilon)}{1 + \epsilon}$		$\frac{R_A \epsilon}{1 + \epsilon}$
R_A, Δh	$R_A - \Delta h$	$\sqrt{R^2_A - 2R_A \Delta h}$		$R_A - 2\Delta h$	$\frac{\Delta h}{R_A - \Delta h}$	
R_P, ϵ	$\frac{R_P}{1 - \epsilon}$	$R_P\sqrt{\frac{1 + \epsilon}{1 - \epsilon}}$	$\frac{R_P(1 + \epsilon)}{1 - \epsilon}$			$\frac{R_P \epsilon}{1 - \epsilon}$
R_P, Δh	$R_P + \Delta h$	$\sqrt{R^2_P + 2R_P \Delta h}$	$R_P + 2\Delta h$		$\frac{\Delta h}{R_P + \Delta h}$	
ϵ, Δh	$\frac{\Delta h}{\epsilon}$	$\frac{\Delta h}{\epsilon}\sqrt{1 - \epsilon^2}$	$\frac{\Delta h}{\epsilon} + \Delta h$	$\frac{\Delta h}{\epsilon} - \Delta h$		

TABLE III.—*Conversion Between Geometric and Dynamic Parameters of Elliptic Orbit.*

Geometric Parameters	Dynamic Parameters: Apogee Velocity, V^2_A	Perigee Velocity, V^2_P	Angular Momentum, H^2	Negative of Total Energy, $-E$
R_a, R_b	$K\dfrac{R_a - \sqrt{R^2_a - R^2_b}}{R^2_a + R^2_a\sqrt{R^2_a - R^2_b}}$	$K\dfrac{R_a + \sqrt{R^2_a - R^2_b}}{R^2_a - R^2_a\sqrt{R^2_a - R^2_b}}$	$K\dfrac{R^2_b}{R_a}$	
R_a, R_A	$\dfrac{K(2R_a - R_A)}{R_aR_A}$	$\dfrac{KR_A}{R_a(2R_a - R_A)}$	$K\left(\dfrac{2R_aR_A - R^2_A}{R_a}\right)$	
R_a, R_P	$\dfrac{K}{R_a}\left(\dfrac{R_P}{2R_a - R_P}\right)$	$\left(\dfrac{K(2R_a - R_P)}{R_aR_P}\right)$	$K\left(\dfrac{2R_PR_a - R^2_P}{R_a}\right)$	$\dfrac{K}{2R_a}$
R_a, ϵ	$\dfrac{K}{R_a}\left(\dfrac{1-\epsilon}{1+\epsilon}\right)$	$\dfrac{K}{R_a}\left(\dfrac{1+\epsilon}{1-\epsilon}\right)$	$KR_a(1-\epsilon^2)$	
R_a, Δh	$\dfrac{K}{R_a}\left(\dfrac{R_a - \Delta h}{R_a + \Delta h}\right)$	$\dfrac{K}{R_a}\left(\dfrac{R_a + \Delta h}{R_a - \Delta h}\right)$	$K\left(\dfrac{R^2_a - \Delta h^2}{R_a}\right)$	
R_b, R_A	$\dfrac{K}{R_A}\left(\dfrac{2R^2_b}{R^2_b + R^2_A}\right)$	$\dfrac{2K}{R^2_b}\left(\dfrac{R^2_A}{R^2_A + R^2_b}\right)$	$K\left(\dfrac{2R_AR^2_b}{R^2_A + R^2_b}\right)$	$\dfrac{KR_A}{R^2_A + R^2_b}$
R_b, R_P	$\dfrac{R^2_P}{R^2_b}\left(\dfrac{2KR_P}{R^2_P + R^2_b}\right)$	$\dfrac{2KR^2_b}{R_P(R^2_P + R^2_b)}$	$K\left(\dfrac{2R_PR^2_b}{R^2_b + R^2_P}\right)$	$\dfrac{KR_P}{R^2_P + R^2_b}$
R_b, ϵ	$\dfrac{K\sqrt{1-\epsilon^2}}{R_b}\left(\dfrac{1-\epsilon}{1+\epsilon}\right)$	$\dfrac{K\sqrt{1-\epsilon^2}}{R_b}\left(\dfrac{1+\epsilon}{1-\epsilon}\right)$	$K\left(\dfrac{R_b(1-\epsilon^2)}{\sqrt{1-\epsilon^2}}\right)$	$\dfrac{K\sqrt{1-\epsilon^2}}{2R_b}$

TABLE III.—*Continued*

Geometric Parameters	Dynamic Parameters			
	Apogee Velocity, V^2_A	Perigee Velocity, V^2_P	Angular Momentum, H^2_B	Negative of Total Energy, $-E_B$
R_b, Δh	$\frac{K}{\sqrt{R^2_b + \Delta h^2}}\left(\frac{\sqrt{R^2_b + \Delta h^2} - \Delta h}{\sqrt{R^2_b + \Delta h^2} + \Delta h}\right)$	$\frac{K}{\sqrt{R^2_b + \Delta h^2}}\left(\frac{\sqrt{R^2_b + \Delta h^2} + \Delta h}{\sqrt{R^2_b + \Delta h^2} - \Delta h}\right)$	$K\frac{R^2_b}{\sqrt{R^2_b + \Delta h^2}}$	$\frac{K}{2\sqrt{R^2_b + \Delta h^2}}$
R_A, R_P	$\frac{2KR_P}{R_A(R_P + R_A)}$	$\frac{2KR_A}{R_P(R_A + R_P)}$	$K\left(\frac{2R_AR_P}{R_A + R_P}\right)$	$\frac{K}{R_A + R_P}$
R_A, ϵ	$\frac{K}{R_A}(1 - \epsilon)$	$\frac{K}{R_A}\frac{(1 + \epsilon)^2}{(1 - \epsilon)}$	$KR_A(1 - \epsilon)$	$\frac{K(1 + \epsilon)}{2R_A}$
$R_a\Delta h$	$\frac{K}{R_A}\left(\frac{R_A - 2\Delta h}{R_A - \Delta h}\right)$	$\frac{KR_A}{(R_A - \Delta h)(R_A - 2\Delta h)}$	$KR_A\left(\frac{R_A - 2\Delta h}{R_A - \Delta h}\right)$	$\frac{K}{2(R_A - \Delta h)}$
R_P, ϵ	$\frac{K}{R_P}\frac{(1 - \epsilon)^2}{(1 + \epsilon)}$	$\frac{K}{R_P}(1 + \epsilon)$	$KR_P(1 + \epsilon)$	$\frac{K(1 - \epsilon)}{2R_P}$
R_P, Δh	$\frac{KR_P}{(R_P + \Delta h)(R_P + 2\Delta h)}$	$\frac{K}{R_P}\frac{(R_P + 2\Delta h)}{(R_P + \Delta h)}$	$KR_P\left(\frac{R_P + 2\Delta h}{R_P + \Delta h}\right)$	$\frac{K}{2(R_P + \Delta h)}$
ϵ, Δh	$\frac{K\epsilon}{\Delta h}\left(\frac{1 - \epsilon}{1 + \epsilon}\right)$	$\frac{K\epsilon}{\Delta h}\left(\frac{1 + \epsilon}{1 - \epsilon}\right)$	$K\frac{\Delta h}{\epsilon}(1 - \epsilon^2)$	$\frac{K\epsilon}{2\Delta h}$

from which

$$\frac{r(t)}{R_a} = 1 + \frac{\epsilon^2}{2} - \left(\epsilon - \frac{3}{8}\epsilon^3\right)\cos \omega_0 t - \left(\frac{1}{2}\epsilon^2 - \frac{1}{3}\epsilon^4\right)\cos 2\,\omega_0 t - \frac{3}{8}\epsilon^3 \cos 3\,\omega_0 t + 0(\epsilon^4). \quad (12)$$

Note that a series for $(R_a/r)^2$ in terms of time can be obtained easily from Eq. 10 using the fact that $(R_a/r)^2 = d\beta^*/d(\omega_0 t)$.

A few other useful expressions from (3) are included for reference.

$$R_a/r = 1 + \sum_{n=1}^{\infty} J_n(n\epsilon)\cos n\,\omega_0 t \quad (13)$$

$$(r/R_a)^2 = 1 + \frac{3}{2}\epsilon^2 - 4\sum_{n=1}^{\infty}\frac{1}{n^2}J_n(n\epsilon)\cos n\,\omega_0 t, \quad (14)$$

where J_n is the Bessel function of the first kind of order n. Also, series expansions for $\sin \beta^*$ and $\cos \beta^*$ can be obtained from the relationships

$$\epsilon \cos \beta^* = (1 - \epsilon)(R_a/r) - 1 \quad (15)$$

$$\epsilon \sin \beta^* = \sqrt{1 - \epsilon^2}\, d(r/R_a)/d\omega_0 t. \quad (16)$$

Finally, it often is convenient to have expressions for the velocity, its components, and its inclination to the local horizontal as functions of β^* and r:

$$V^2 = K(1 + \epsilon^2 + 2\epsilon \cos \beta^*)/R_a(1 - \epsilon^2) \quad (17)$$

$$V_H = \sqrt{\frac{K}{R_a(1 - \epsilon^2)}}\,(1 + \epsilon \cos \beta^*) \quad (18)$$

$$V_V = \sqrt{\frac{K}{R_a(1 - \epsilon^2)}}\,\epsilon \sin \beta^* \quad (19)$$

$$\tan \psi = \epsilon \sin \beta^*/(1 + \epsilon \cos \beta^*). \quad (20)$$

2.3 Rationale of Perturbation Analysis

The most important assumption under which the orbit problems of this paper are treated is that the various perturbation effects are linearly additive. Neither second-order results for individual sources nor the cross-coupling of the physical effects is included. However, in nonlinear systems such as those of celestial mechanics one generally cannot simply superimpose the responses to a number of forces in finding the response to their sum. Therefore the question arises as to the range of validity of the results.

Unfortunately, a quantitative answer cannot be given. However, Poincaré's generalization of Cauchy's existence theorem for analytic ordinary differential equations guarantees that the solution in the presence of many small perturbations is simply the sum of the separate responses to the perturbations, plus terms of higher order in the perturbation parameters. Provided these parameters are sufficiently small, this expansion of the solution is known to converge. Furthermore, for sufficiently small parameter values, the first-order terms describe the motion accurately over any preassigned length of time, even if secular terms are present. However, one is not free to make these parameters arbitrarily small, for they are set by the physical situation. It follows that no estimate of the accuracy with which the first-order terms represent the solution can be obtained without evaluating the higher-order terms and determining their magnitudes. Not even convergence can be proved, in general.

The present analysis is designed to satisfy the needs of artificial satellite system engineering and does not purport to represent a complete and satisfactory solution to the orbit problems purely as problems of celestial mechanics. Therefore, as a practical matter, I feel that the philosophy should be adopted that the important qualitative aspects of the motion all arise in the first-order terms, and that the higher-order terms function principally to modify the numerical values.[9] This has been used as a working hypothesis in the absence of investigation of higher-order and cross-coupling terms.

The separation of perturbing effects is one facet of the rationale of the analysis. Another is the periodicity of solutions. In the presence of most perturbations, orbits are not closed, not periodic. However, at least for nondissipative perturbations, it often is possible to choose a suitably rotating reference frame with respect to which the motion appears to be periodic. This approach, the search for such a reference frame, is used here because of the easy visualization it provides as a picture of the orbit.

However, a subtlety arises which must be noted. Not all of the perturbation effects mentioned above are analyzed relative to the same coordinate system. Even though the motion in each individual case may be periodic when referred to its own reference frame, does it follow that a single system can be chosen with respect to which the combined motion is periodic?

This question is answered affirmatively by the following theorem:

[9] An interesting extension of the present work would be the consideration of some of the first cross-coupling terms to see whether any new qualitative features arise. It certainly is possible that these or the second-order effects may be quantitatively significant as well. For example, it is stated in (3), p. 247, that nearly half of the motion of the lunar perigee is obtained only in the second approximation. The present case may be simpler because of the presence of dominant perturbations (the earth's oblateness and air drag) which tend to overshadow all other effects.

Consider the small perturbations from a number of nondissipative sources. Suppose that reference frames with suitable vector angular velocities can be found such that the motion under each individual source acting alone is periodic as seen from the corresponding reference frame. Then the motion under the combined effects of these perturbation sources is periodic, at least to first-order terms in the small perturbation parameters, as seen from a reference frame whose angular velocity is the vector sum of those corresponding to the individual sources.

To prove this, let the motion be described relative to the orbital plane of Fig. 3 which rotates in inertial space with the angular velocity $\bar{\omega}$. Suppose that this angular velocity is resolved into the $x\ y\ z$ coordinate system by

$$\bar{\omega} = \omega_x \bar{e}_x + \omega_y \bar{e}_y + \omega_z \bar{e}_z, \tag{21}$$

where the $\bar{e}$ are the unit vectors along the axes indicated by the subscripts. The components of $\bar{\omega}$ may be functions of the variables β, t and certain parameters such as γ and the characteristics of the perturbation sources.

The kinetic energy of the satellite is[10]

$$T = \frac{m}{2}\left[\dot{r}^2 + r^2(\dot{\alpha} - \omega_y)^2 + r^2(\dot{\beta}\cos\alpha + \omega_z \cos\alpha - \omega_x \sin\alpha)^2\right]. \tag{22}$$

In general the potential function in the presence of a number of perturbing forces can be written

$$V = -\frac{mK}{r}\left[1 + \nu_1 f^{(1)}(r, \alpha, \beta, t) + \nu_2 f^{(2)}(r, \alpha, \beta, t) + \cdots\right] \tag{23}$$

where $\nu_1, \nu_2, \cdots$ are small parameters and $f^{(1)}, f^{(2)}, \cdots$ are functions whose exact forms need not be specified. Attention is specifically restricted to nondissipative cases for which such a potential function exists, thereby leaving air drag out of consideration.

The equations of motion in r, α and β become (the independent variable being t)

$$\ddot{r} = r\left[(\dot{\alpha} - \omega_y)^2 + (\dot{\beta}\cos\alpha + \omega_z \cos\alpha - \omega_x \sin\alpha)^2\right] - \frac{K}{r^2}\left\{1 + \nu_1\left[f^{(1)} - r\frac{\partial f^{(1)}}{\partial r}\right] + \nu_2\left[f^{(2)} - r\frac{\partial f^{(2)}}{\partial r}\right] + \cdots\right\} \tag{24}$$

$$\frac{d}{dt}\left[r^2(\dot{\alpha} - \omega_y)\right] = -r^2\left[\dot{\alpha}\frac{\partial \omega_y}{\partial \alpha} + (\dot{\beta}\cos\alpha + \omega_z \cos\alpha - \omega_x \sin\alpha) \times (\dot{\beta}\sin\alpha + \omega_z \sin\alpha + \omega_x \cos\alpha)\right] + \frac{K}{r}\left[\nu_1\frac{\partial f^{(1)}}{\partial \alpha} + \nu_2\frac{\partial f^{(2)}}{\partial \alpha} + \cdots\right] \tag{25}$$

[10] The case $\omega_x = \dot{\Lambda}_\Omega \sin\gamma \sin\beta$, $\omega_y = \dot{\Lambda}_\Omega \sin\gamma \cos\beta$, $\omega_z = \dot{\Lambda}_\Omega \cos\gamma$ is the one considered in Chapter 3.

$$\frac{d}{dt}\left[r^2\cos\alpha(\dot\beta\cos\alpha+\omega_z\cos\alpha-\omega_x\sin\alpha)\right]=r^2\Bigg[-\dot\alpha\frac{\partial\omega_y}{\partial\beta}$$

$$+(\dot\beta\cos\alpha+\omega_z\cos\alpha-\omega_x\sin\alpha)\left(\frac{\partial\omega_z}{\partial\beta}\cos\alpha-\frac{\partial\omega_x}{\partial\beta}\sin\alpha\right)\Bigg]$$

$$+\frac{K}{r}\left[\nu_1\frac{\partial f^{(1)}}{\partial\beta}+\nu_2\frac{\partial f^{(2)}}{\partial\beta}+\cdots\right]. \quad (26)$$

In applying a perturbation method to these equations, one assumes

$r=r_0+\nu_1 r+\nu_2 r+\cdots$ terms in $\nu_3, \nu_4, \cdots$ and products and powers of the ν_i (27)

$$\alpha=\nu_1\alpha_1+\nu_2\alpha_2+\cdots \quad (28)$$

$$\beta=\beta_0+\nu_1\beta_1+\nu_2\beta_2+\cdots \quad (29)$$

$$\omega_x=\nu_1 X_1+\nu_2 X_2+\cdots \quad (30)$$

$$\omega_y=\nu_1 Y_1+\nu_2 Y_2+\cdots \quad (31)$$

$$\omega_z=\nu_1 Z_1+\nu_2 Z_2+\cdots \quad (32)$$

The zero order equations are simply the equations for elliptic motion,

$$\ddot r_0=r_0\dot\beta^2-\frac{K}{r_0} \quad (33)$$

$$\frac{d}{dt}(r_0^2\dot\beta)=0. \quad (34)$$

The first order equations are, with $i=1, 2, \cdots$

$$\ddot r_i=r_i\dot\beta_0^2+2\dot\beta_0 r_0(\dot\beta_i+Z_i)-\frac{K}{r_0}\left[\alpha f^{(i)}-r_0\frac{\partial f^{(i)}}{\partial r}-\frac{2r_i}{r_0}\right] \quad (35)$$

$$\frac{d}{dt}\left[r_0^2(\dot\alpha_i-Y_i)\right]=-r_0^2\dot\beta_0(\dot\beta_0\alpha_i+X_i)+\frac{K}{r_0}\frac{\partial f^{(i)}}{\partial\alpha} \quad (36)$$

$$\frac{d}{dt}\left[r_0^2(\dot\beta_i+Z_i)+2r_0\dot\beta_0 r_i\right]=r_0^2\dot\beta_0\frac{\partial Z_i}{\partial\beta}+\frac{K}{r_0}\frac{\partial f^{(i)}}{\partial\beta}. \quad (37)$$

To say that there exists a rotating frame with respect to which the motion appears periodic when a particular perturbation source (say the k^{th}) is considered, is equivalent to selecting an X_k, Y_k and Z_k such that the solutions of Eqs. 35–37 for $i=k$ are periodic. Suppose that this can be done for all sources considered. Then X_i, Y_i, Z_i are deter-

mined uniquely for all i; and, because of the way in which these quantities enter Eqs. 35–37, the choice to make the motion appear periodic under one perturbation does not affect the choice for any other perturbation source.

That is, it is possible to choose X_i, Y_i, Z_i ($i = 1, 2, \cdots$) such that there is simultaneous periodicity for all sources. This uniquely specifies the angular velocity of the single coordinate reference frame, making it in view of Eqs. 30–32 equal to

$$\bar{\omega} = \bar{e}_x \left(\sum_i \nu_i X_i \right) + \bar{e}_y \left(\sum_i \nu_i Y_i \right) + \bar{e}_z \left(\sum_i \nu_i Z_i \right) + \text{higher order terms in the } \nu_i \quad (38)$$

and the theorem is proved.

2.4 *Analysis of Minor Effects*[11]

A number of sources of perturbing force can be conceived to act on a close artificial satellite. These include:

1. oblateness of the earth
2. air drag
3. sun, moon, and planets
4. asymmetries of the earth (deviations of the geoid from an ellipsoid or revolution)
5. tidal friction
6. attitude motions of the satellite
7. electromagnetic fields
8. relativistic effects

Of these, the first and second are dominant effects, while the remainder are minor.

Relative to the perturbation caused by the oblateness of the earth, it is shown later that the orbit appears closed and periodic relative to a suitably rotating frame of reference. In particular, the orbit appears to remain near a plane which rotates about the earth's polar axis at a rate which depends on the inclination of the plane. This effect is called the "regression of the nodes." Moreover, the line of apses of the generating ellipse rotates slowly in the regressing plane, the effect being called the "precession of the perigee."

The other dominant perturbation, air drag, eventually brings any close satellite to earth. It is the only significant dissipative perturbation. As mentioned previously, it is analyzed elsewhere. The method

[11] Since this section was originally written, these effects have received considerable verbal and written discussion. I do not believe, though, that this has touched upon the present material on tidal friction. I have separately given a method for analyzing the air drag effect, as (13), in which some other references to this topic are given.

of variation of parameters is used, and approximate expressions are obtained for the decay of radius and eccentricity with time, and for the gradual change of orbital frequency.

Consider now the minor perturbations.

An obvious source of orbital perturbation is the presence of gravitating bodies other than the earth. Spitzer (12) has summarized the effects of sun and moon, showing that changes of radius are insignificant and that orbital regression effects added have periods of the order of thousands of years, so that changes over one year are small. Maximum changes in the longitude of the ascending node are about 10 min. and 4 min. from moon and sun, respectively. It is estimated that fluctuations of orbital inclination do not exceed about 30 sec. and 10 sec. from moon and sun, respectively.

It is concluded that the effects of the sun and moon (and, even more so, of the planets) can be ignored over periods of the order of a year. It is possible that they can be observed over such a period, if not obscured by the oblateness and air drag perturbations, but they will not be operationally significant for most satellite applications.

In analyzing the effects of its oblateness, the earth is assumed to have rotational symmetry about its polar axis. This is not quite true. The earth is a relatively rigid body with three principal axes of inertia. As one of these corresponds closely to the polar axis, the other two must be approximately in the equatorial plane. If the earth's moments of inertia about these two axes are equal, the satellite motion remains independent of the earth's rotation. However, there is some evidence that there is a difference in the moments about the two equatorial principal axes. A term should therefore be inserted into the potential function of the earth which depends on the satellite longitude relative to these axes.

According to (11) the coefficient of this longitude-dependent term is two orders of magnitude below that of the first-order oblateness term. Jeffreys (5) remarks that this effect can be analyzed satisfactorily by a first-order perturbation procedure, if analysis is desired, but that the effect of asymmetry, unlike that of oblateness, has never been observed in the motion of the moon. Accordingly, it is conjectured that the earth's rotational asymmetry is relatively unimportant for a small, close satellite over a period of time of the order of one year.

Tides raised by the satellite and their corresponding dissipative effect are insignificant, contrary to the claim of (7). It is worth investigating why this claim is incorrect.[12] The equation which expresses the conservation of angular momentum for the earth-satellite system (for a quasi-circular orbit of radius R_a) is

$$H_E + mR_a^2\omega_0 = \text{constant}, \tag{39}$$

[12] The author is idebted to Dr. John V. Breakwell for this argument.

where H_E is the spin angular momentum of the earth and ω_0 is given by Eq. 6.

Another equation must be added to Eq. 39 to express the effect of tidal friction. With ω_0 greater than the spin angular velocity Ω of the earth, the gravitational tide raised on the earth by the satellite may be expected to lag behind the satellite because of tidal friction. This causes a gravitational torque tending to increase the earth's angular momentum and decrease that of the satellite.[13] The amount of the retardation torque is not known, but the resulting torque on the earth is assumed in (7) to be

$$\frac{dH_E}{dt} = C(\omega - \omega_1)(\omega - \omega_2), \tag{40}$$

where ω_1 and ω_2 are the real solutions of

$$\frac{H_E\omega_0}{\Omega} + \frac{mK^2}{\omega_0^2} = H_E + \frac{mK^2}{\Omega}. \tag{41}$$

That is, ω_1 and ω_2 are the two possible satellite angular velocities which are equal to the angular velocity of the earth.

It is in the value of C that the fallacy of (7) arises. There, it is stated that "since the torque is a gravitational effect," C must be proportional to the mass m causing it, and "since [the torque] depends on the variation of gravitational pull with distance," C must also be proportional to mR_a^{-3}. However, the following is shown to be the case in (5, p. 220). First, the satellite raises a tide of height proportional to mR_a^{-3}. Second, the torque exerted on the earth is proportional to mR_a^{-3}. Thus it must be that C is proportional to $(mR_a^{-3})^2$ rather than to mR_a^{-3}.

With this correction, the theory of (7) estimates that the time of descent from 800 km., a typical value, would be of an order exceeding 10^{12} year.

Consider next the effect of attitude motions of the satellite. If it is considered to be a distributed rigid body rather than a point mass, the equations of motion of the center of mass around the earth are generally coupled to the equations of attitude motion about the center of mass.

This effect is equivalent to introducing into the potential function for orbital motion a perturbation term whose coefficient (analogous to the coefficient μ of the oblateness perturbation discussed in Section 4) is $(I - I_p)/ma^2$, where $I - I_p$ is the difference between the satellite moments of inertia about a diametral axis and axis of rotational symmetry. For a typical satellite vehicle this coefficient is many orders of magnitude smaller than μ.

[13] This is the reverse of the effect between earth and moon because the angular velocity of the moon is less than that of the earth and the tidal bulge preceded the position of the moon.

It is concluded that the effect is completely negligible and that orbital motions can be analyzed independently of attitude motions. The converse is also true, that attitude motions can be analyzed as if the orbital motions resulted in purely external exciting forces.

An artificial satellite will have a magnetic moment and a charge distribution. The magnetic moment arises from permanent magnetic fields originating in internal equipment and, by induction, from the magnetic field of the earth. A net electrical charge may exist from ion collection during the ascent through the atmosphere, and a charge distribution is induced by the electric field of the earth. Both torques and body forces result from these effects.

The analysis of the interaction of the earth's electric and magnetic fields with the satellite's field is an exceptionally difficult problem. A part of this is purely formal analytical difficulty, but a more troublesome fact is that the satellite field is quite sensitive to its configuration and the precise nature of its electric and magnetic properties and its internal equipment.

Therefore, I am unable to make a general statement about the magnitude of the electromagnetic interaction effect. However, some crude estimates of upper bounds have led me to conjecture that the force from this source normally would be entirely negligible compared to the perturbation force from the oblateness of the earth.

Relativistic effects, as might be expected, are insignificant relative to the oblateness effect. Perturbation terms are obtained in the relativistic equations of motion which have a ratio to the usual inverse square term closely equal to $3R_a^2\omega_0^2/c^2$, where c is the velocity of light. For altitudes of the order of hundreds of miles, this ratio is of the order of 10^{-9}, which is to be compared with the corresponding coefficient 5×10^{-4} for the oblateness perturbations.

(*To be continued*)

REFERENCES

(1) LEON BLITZER, MORRIS WEISFELD AND A. D. WHEELON, "Perturbations of a Satellite's Orbit Due to the Earth's Oblateness," *J. Appl. Phys.*, Vol. 27, pp. 1141–1149 (1956).

(2) DIRK BROUWER, "The Motion of a Particle with Negligible Mass Under the Gravitational Attraction of a Spheroid," *Astronomical J.*, Vol. 51, p. 223–231 (1956).

(3) E. W. BROWN, "Lunar Theory," Cambridge, Cambridge University Press, 1896.

(4) JOHN DE NIKE, "The Effect of the Earth's Oblateness and Atmosphere on a Satellite Orbit," in "Earth Satellites as Research Vehicles," Monograph 2, *Journal of The Franklin Institute* series, 1956, pp. 79–86.

(5) HAROLD JEFFRIES, "The Earth, Its Origin, History and Physical Constitution," Second Edition, Cambridge, Cambridge University Press, 1929.

(6) H. G. L. KRAUSE, "Die Sakularstörungen einer Aussenstationsbahn," in "Probleme aus der Astronautischen Grundlagenforschung," edited by H. H. Kolle (Vortrage III Internationalen Astronautischen Kongress, Stuttgart, 1–6 September 1952), pp. 162–173.

(7) J. LOGIE, "Effect of Tidal Friction on a Near Satellite," *J. Brit. Interplanetary Soc.*, Vol. 13, p. 170–175 (1954).

(8) W. D. MACMILLAN, "Periodic Orbits About an Oblate Spheroid," Chapter IV of "Periodic Orbits," by F. R. Moulton, *et al*, Washington, D. C., Carnegie Institution, 1920.

(9) F. R. MOULTON, "Celestial Mechanics," New York Macmillan Company, 1902.

(10) N. V. PETERSEN, "Lifetimes of Satellites in Near-Circular and Elliptic Orbits," *Jet Propulsion*, Vol. 26 pp. 341–351, 368 (1956).

(11) Smithsonian Physical Tables, Eighth Edition. Smithsonian Institute, Washington, D. C., 1934.

(12) LYMAN SPITZER, JR., "Perturbations of a Satellite Orbit," *J. Brit. Interplanetary Soc.*, Vol. 9, pp. 131–136 (1950).

(13) R. E. ROBERSON, "Effect of Air Drag on Elliptic Satellite Orbits," Am. Rocket Soc. paper 466–57, presented at the ARS Semi-Annual Meeting, San Francisco, June 12, 1957.

ANALYSIS OF PULSE PILE-UP EFFECTS IN A PULSE-COUNTING SYSTEM *

BY

RICHARD H. FRAZIER [1]

ABSTRACT

In a pulse-counting system such as illustrated in the block diagram of Fig. 1, the counting device may give spurious counts caused by the pile-up or superposition of small pulses originated by background radiation, so that the equivalent of a pulse characteristic of the radiation under examination is approximated. This paper analyzes the rate of spurious counts as influenced by the average rate of occurrence of the small background pulses, the discriminator setting in terms of the height of such pulses, the resolving time of the amplifier-discriminator combination, and the resolving time of the counting device.

I. STATEMENT OF THE PROBLEM

The problem studied relates to the counting of pulses due to radiation a measurement of which is desired in the presence of a background of smaller pulses due to unwanted radiation. Though the unwanted pulses may in general be much smaller individually than the wanted pulses, the unwanted pulses may occur with such frequency that occasionally sufficient overlap or "pile-up" may develop to give a resultant pulse height comparable to the height of wanted pulses and a spurious count may result. An estimate of the rate of these spurious counts for comparison with the desired counting rate for wanted pulses, together with some understanding of the manner in which counts develop in the counting system, should be of some assistance in the design and operation of the system to minimize such spurious counts.

The type of pulse-counting system studied consists essentially of a chamber in which ionization is created either directly or indirectly by the radiation to be measured, an amplifier that enlarges the pulses generated by ionization in the chamber, a discriminator that recognizes only those enlarged pulses that exceed a predetermined voltage level and is triggered by those pulses to deliver still larger and essentially uniform rectangular pulses to a scaler and register or other counting device. The simple block diagram of Fig. 1 indicates the sequence of apparatus. This paper does not give any treatment of the details of the circuits or devices that may be involved in the system but aims merely to give sufficient de-

* The work on which this paper is based was performed as Consultant to the Bettis Plant, Pittsburgh, Pa., operated for the United States Atomic Energy Commission by the Westinghouse Electric Corporation. Permission to publish the material has been granted by the Atomic Energy Commission.

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

scription of the action of the system to provide basis for general analysis.

The voltage height, shape and duration of a pulse delivered by the chamber to the amplifier depends not only on the energy of the ionizing particle, the kind of gas in the chamber plus its temperature and pres-

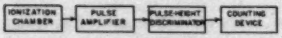


FIG. 1. Block diagram of pulse-counting system.

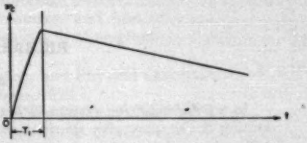


FIG. 2. Pulse delivered to amplifier.

sure, and the polarizing voltage, but depends also upon the geometry of the chamber, the direction of the line along which ionization occurs, and the manner of coupling the chamber to the amplifier, as discussed by W. W. Ramage and R. H. Frazier in another paper (1).[2] The form of pulse delivered by the chamber to the amplifier may be essentially as illustrated in Fig. 2, which is for a parallel-plate chamber with the line of ionization parallel to the plates.[3] In passing through the amplifiers the pulse height, shape, and duration are further modified by the parameters of the vacuum tubes, interstage coupling and output networks, so that an amplified pulse of voltage v_a having the general form of Fig. 3 is delivered by the amplifier to the discriminator, which in turn delivers a rectangular pulse of height V_0 such as shown in Fig. 4 to the counting

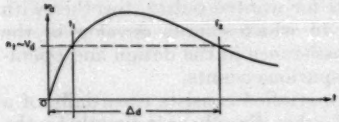


FIG. 3. Pulse delivered to discriminator.

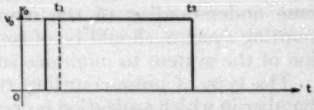


FIG. 4. Pulse delivered to counting device.

device. The time constants of the input, coupling, and output networks for the amplifier must be adjusted so that for the average frequency with which pulses occur, voltages on capacitors cannot build up sufficiently to block the operation of the vacuum tubes. This situation is one aspect of pulse pile-up, though not the problem of concern in this paper.

The assumption in Fig. 3 is that a second pulse does not originate until some time beyond t_2. The time interval Δ_d, counted from the origin of a pulse in the ionization chamber to the time t_2 at which the amplified

[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] Reference (1), p. 28. Figure 2 and Figs. 3 and 4 that follow are not drawn to any particular scale, either indivudally or relative to each other.

pulse falls to the discriminator setting V_d, is called the *resolving time* of the amplifier and discriminator, and is an important quantity in the analysis that follows. No other pulse that originates during that interval is counted. Another pulse that originates within Δ_d merely extends the dead time of the discriminator beyond t_2. If pulses originate at too high a frequency the discriminator may become completely paralyzed. On the other hand, when the interval between pulses is large compared with Δ_d, amplified pulses that do not exceed V_d are not counted, but occasionally two or more pulses may originate within Δ_d each of which when amplified may not exceed V_d individually, but the overlapped pulses superposed may exceed V_d and cause a count. This situation is another aspect of "pile up," closely related to the problem treated herein.

When the discriminator is triggered at t_1, Fig. 3, it may deliver to the counting device a rectangular pulse that extends to t_2, Fig. 4, or the pulse may be "clipped" as shown by the dotted line. Such pulses or impulses do not necessarily represent counts but rather should be regarded as *requests* made by the discriminator to the counting device to count. The counting device itself requires a certain minimum time to complete its operation after it has received a request from the discriminator, which may be called a resolving time Δ_c. Hence when a request has been presented to the counting device, another request made within Δ_c cannot be granted, and whether or not the dead time of the counting device is extended by requests made with Δ_c is a matter of design.

The height of the pulses delivered by the chamber to the first stage of the amplifier, sometimes called the preamplifier, may be exceedingly small, of the order of 10^{-4} volt or less, with a rise time T_1, Fig. 2, around 10^{-6} sec. or less. Hence the problem of distinguishing these pulses above the general level of noise disturbances of various kinds is considerable. These noise disturbances may originate either within the system of Fig. 1 or outside of it. The noise that originates within the system arises from the thermal noise of resistors and from the various noises associated with electron emission and convection in the vacuum tubes, or from unwanted radiation excited within the chamber directly or indirectly by the radiation under measurement. The principal troubles from thermal and vacuum-tube noises arise from the noises that originate in the first stage of the amplifier, or the preamplifier, because such noises that originate in succeeding stages generally are negligible in comparison with the amplified pulse that reaches those stages. However, the noises that originate in the first stage are amplified also, so that the problem of achieving the largest pulse height at the output of the amplifier in comparison with the amplifier noise delivered there involves a study of tube selection and operating points in association with the input, coupling, and output networks of the amplifier so as to generate as little

noise as possible and then to pass as little of it as possible without undue attenuation of the pulse. This problem has been studied extensively and in considerable detail by A. B. Gillespie (2, 3, 4), and by W. W. Ramage and R. H. Frazier.[4] Noise that originates outside the system may arise from stray electromagnetic fields against which the system must be shielded, or from unwanted radiation the particles of which penetrate the chamber and perhaps also originate secondary radiations within the chamber. This paper presents a study of the influence of the pulses that originate from unwanted radiation when they overlap so that the amplified resultant pulse height at the discriminator exceeds V_d, Fig. 3, and a spurious count is registered.

II. METHOD OF ATTACK

The problem in mind in association with this analysis was the counting of pulses initiated by neutron events in the presence of a background of smaller pulses initiated by alpha events that might occur with sufficient frequency to overlap or pile up at intervals and give resultant pulses at the input to the discriminator high enough to cause spurious counts. The origin of alpha events is taken to occur in accordance with Poisson distribution in time, which assumption seems to be generally accepted as valid. According to such distribution, the probability that n events originate within a short time inverval Δ is

$$P_n = \frac{(\bar{n}\Delta)^n}{n!}\epsilon^{-n\Delta} \tag{1}$$

in which $\bar{n}$ is the average rate of events taken over a very long time. If all pulse heights are alike and Δ is sufficiently small, the resultant pulse height due to pile-up of n pulses is essentially proportional to n. Then if the time interval Δ is taken essentially equal to the resolving time of the amplifier plus discriminator Δ_d, some threshold number of pulses n_i gives sufficient pile-up that just fails to exceed the discriminator setting V_d; that is, for

$$n > n_i \tag{2}$$

the discriminator setting V_d is exceeded, and for

$$n \leq n_i \tag{3}$$

the discriminator setting is not exceeded, and as indicated on Fig. 3, n_i is nearly proportional to V_d. This simplified view ignores variation in shape and height among the n pulses involved, which variation has less and less influence as n is increased and Δ is decreased.

The probability that n_i or fewer events originate within time interval

[4] Informal reports from which a paper is in preparation.

Δ is

$$P_- = \left[1 + \frac{\bar{n}\Delta}{1!} + \frac{(\bar{n}\Delta)^2}{2!} + \cdots + \frac{(\bar{n}\Delta)^{n_i}}{n_i!}\right] \epsilon^{-\bar{n}\Delta} \tag{4}$$

and the probability that more than n_i events originate within time interval Δ is

$$P_+ = 1 - P_-. \tag{5}$$

If time is divided into equal successive intervals Δ, Fig. 5, the probable

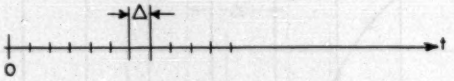


FIG. 5. Time divided into consecutive equal intervals.

rate of occurrence of intervals within which more than n_i events originate is

$$D_i = P_+/\Delta, \tag{6}$$

an illustrative plot of which is Fig. 6 (for $\bar{n}\Delta = 10$, which makes computation easy and plotting to a reasonable scale feasible). Curves for other values of $\bar{n}\Delta$ have the same form but are compressed or elongated along the n_i axis.

This function, P_+/Δ, or the substantial equivalent, sometimes is taken as the counting rate for spurious pulses. The interpretation is as follows. The probability that exactly n events originate within an interval Δ, Eq. 1, is a maximum for

$$n = \bar{n}\Delta, \tag{7}$$

and for the conditions of Fig. 6, the maximum probability that exactly n_i events originate within the time interval Δ is for $n_i = 10$ which is at the maximum slope (point of inflection) of the curve. For lower or higher discriminator settings, (lower or higher n_i) the probability that exactly n_i events originate within time interval Δ is less than for $n_i = \bar{n}\Delta = 10$. For example at a discriminator setting corresponding to $n_i = 1$, all pulses are passed except those due to a single event within time interval Δ, the probability of which is relatively low. Likewise the probability of having only two events within time interval Δ is relatively low, so that elimination of events by discriminator settings corresponding to n_i small compared with $\bar{n}\Delta$ makes little difference in the count. In the region of $n_i = \bar{n}\Delta$ the counting rate changes rapidly with discriminator setting, because as n_i is increased through 10 the probability of eliminating the influence of intervals within which 8, 9, 10, or 11 events originate is relatively high. As the discriminator setting is further

increased, corresponding to n_1 high compared to $\bar{n}\Delta$, the probability of having for example 19 or 20 events originate within time interval Δ is relatively low, so that elimination of those events makes little difference in the count; in other words the discriminator voltage V_d then is set so high that essentially no pulses due to pile-up can exceed V_d. Evidently, according to this argument, the rate of spurious counts can be made as small as desirable by indefinite increase of V_d, but in the process the rate of counts from wanted radiation may be too drastically reduced also.

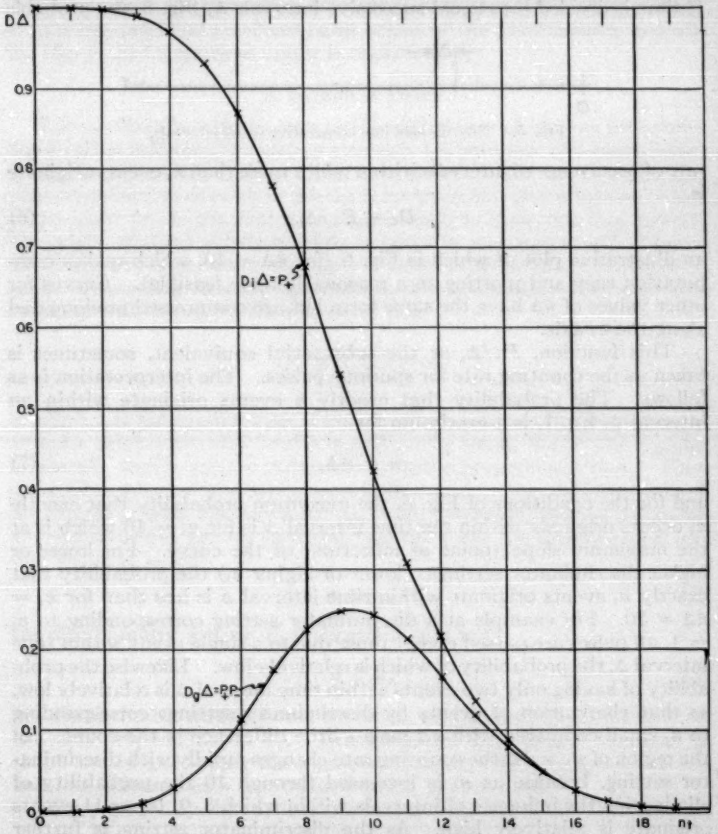


FIG. 6. Plots of $D_1\Delta$ and $D_2\Delta$ for $\bar{n}\Delta = 10$.

Use of P_+/Δ as the counting rate for spurious pulses is a considerable oversimplification and involves substantial errors fully evident even within the limits of the simplifying assumptions relating to pulse shape and height. The counting rate as represented by D_t really should be interpreted only as a rate of requests presented to the counting device, and as a rate of requests only if the intervals Δ within which more than n_t events originate are isolated from each other. Furthermore, within two adjacent time intervals Δ within neither of which more than n_t events originate may be found an equal overlapping time interval Δ', Fig. 7,

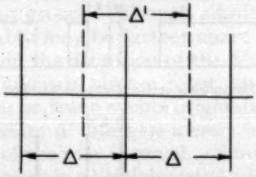


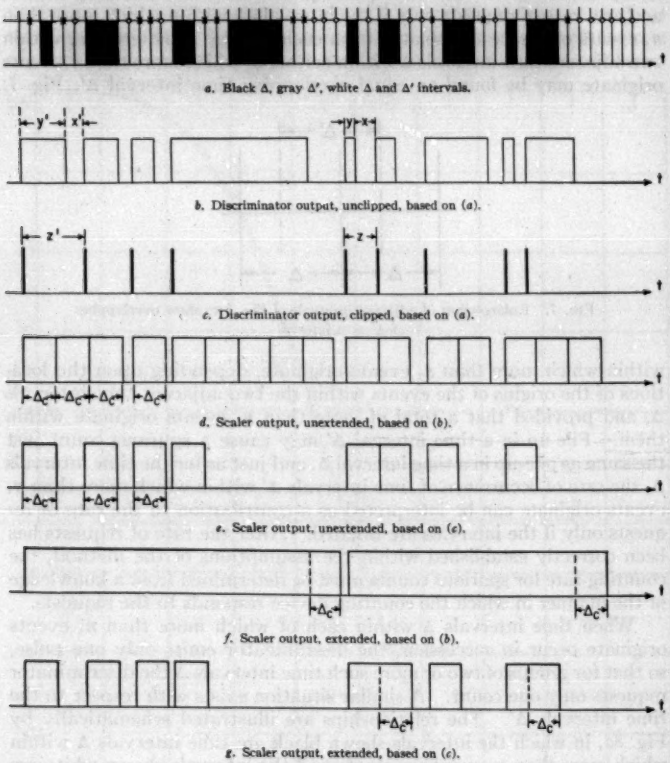
FIG. 7. Enlargement of adjacent intervals of Fig. 5 to show overlapping of $\Delta' = \Delta$ interval.

within which more than n_t events originate, depending upon the locations of the origins of the events within the two adjacent time intervals Δ, and provided that a total of more than n_t events originate within them. Pile-up in a time interval Δ' may cause a spurious count just the same as pile-up in a time interval Δ, and just as for the time intervals Δ, the rate of occurrence of time intervals Δ' within which more than n_t events originate can be interpreted as a contribution to the rate of requests only if the intervals are isolated. After the rate of requests has been correctly established within the assumptions of the method, the counting rate for spurious counts must be determined from a knowledge of the manner in which the counting device responds to the requests.

When time intervals Δ within each of which more than n_t events originate occur in succession, the discriminator emits only one pulse, so that for groups of two or more such time intervals Δ the discriminator requests only one count. A similar situation exists with respect to the time intervals Δ'. The relationships are illustrated schematically by Fig. 8*a*, in which the intervals shown black are time intervals Δ within which more than n_t events originate and the intervals shown white are time intervals Δ within which n_t or fewer events originate, and an "x" at the junction of two of them indicates that they contain an overlapping time interval Δ', Fig. 7, within which more than n_t events originate. Hereafter for brevity these three kinds of time intervals are called black, white, and gray, respectively. Actually the black and the gray intervals should be shown with various heights above a certain level corresponding to n_t, and the white intervals should be shown with various

heights below or equal to that level, but since the diagram is schematic, that complication is omitted.

If only black and white intervals exist, the inference is that the discriminator emits a pulse, or issues a request to the counting device for



a. Black Δ, gray Δ′, white Δ and Δ′ intervals.

b. Discriminator output, unclipped, based on (*a*).

c. Discriminator output, clipped, based on (*a*).

d. Scaler output, unextended, based on (*b*).

e. Scaler output, unextended, based on (*c*).

f. Scaler output, extended, based on (*b*).

g. Scaler output, extended, based on (*c*).

FIG. 8.

each group of black intervals that is separated from the next group of black intervals by at least one white interval. The rate at which these black groups probably occur can be readily computed and represents the rate of requests in the absence of gray intervals. The location of a gray interval with respect to the two white intervals involved cannot be

exactly determined. All that can be determined is the probability that a gray interval exists. Hence two gray intervals that are consecutive (have a white interval in common) may overlap but never can be separated by as much as $\Delta' = \Delta$. Likewise two gray intervals that are not consecutive (do not have a white interval in common) cannot touch but must be separated by less than $2k\Delta = 2k\Delta'$, in which k is an integer, $k > 0$. On the average the overlap or separation of consecutive gray intervals should be much less than the separation of nonconsecutive gray intervals, so that consecutive gray intervals may be considered to be in the same group and nonconsecutive gray intervals may be considered to be in different groups. Though admittedly two consecutive gray intervals conceivably may be farther apart than two nonconsecutive gray intervals within the definition of the Δ' intervals, nevertheless nonconsecutive gray intervals always must involve at least one Δ' interval within which n_t or fewer events originate, and some overlap of consecutive gray and white Δ' intervals merely indicates a crowding of events toward one end of the gray interval. Similar crowding of events toward one end may happen in a black interval adjacent to a white interval, but the manner of definition of these intervals makes overlap impossible. Yet if the intervention of a single white Δ interval is taken as adequate to terminate a group of black intervals, from the point of view of discriminator response, the intervention of a single white Δ' interval should be considered adequate to terminate a group of gray intervals likewise. Furthermore consecutive gray intervals never can involve a white Δ' interval even though in the extreme the consecutive gray intervals may be separated by nearly $\Delta' = \Delta$. Hence to be consistent with the concept of the response of the discriminator to groups of black intervals, the inference is that if only gray and white intervals exist, the discriminator emits a pulse, or issues a request to the counting device for each group of gray intervals that is separated from the next group of gray intervals by at least one white Δ' interval. A white Δ' interval is indicated by an "o" at the junction of two white Δ intervals. The rate at which these gray groups probably occur can be computed, though the procedure is considerably more complicated than the computation for the black groups, and represents the rate of requests in the absence of black intervals. The total rate of requests is the sum of the rates for the black groups and the gray groups, except when black and gray groups are merged. A gray interval cannot touch a black interval but if the gray interval overlaps a white Δ interval adjacent to the black interval the separation of the gray interval from the black interval cannot be as much as $\Delta = \Delta'$ and a full white interval never can intervene. Then to be consistent with the concept of discriminator response to black or gray intervals, the black and gray groups should be considered as merged. In other words at least a full white interval must intervene between black and gray groups to render them distinct from the point of

view of discriminator response. The correct rate of requests, within the assumptions of the method described, therefore is equal to the total rate of occurrence of distinct black, gray, and black-gray groups. The influence of the merging of black and gray groups upon the rate of requests can be computed fairly readily from the information contained in the analyses of the rate of requests anticipated for the black and the gray groups separately. The full results for the rates of request are given in Section III of this paper and the supporting analyses are given in the full report (5), Appendices A and B. The discriminator output pulses corresponding to Fig. 8*a* are shown schematically in Fig. 8*b*, unclipped, and in Fig. 8*c*, clipped. In deriving these discriminator pulses, they are begun or ended at the middle of a white Δ interval when initiated or terminated by a gray interval. The use of P_+/Δ by comparison is excessive to the extent that it fails to recognize the fact that consecutive intervals in which pile-up occurs are not individually identified and translated into separate requests by the discriminator, and is deficient to the extent that it fails to allow for the fact that not all of the time intervals $\Delta = \Delta'$ in which more than n_i events originate are found by investigating only adjacent time intervals Δ, Fig. 5, but that all other possible intervals of length $\Delta = \Delta'$, Fig. 7, must be investigated also.

In making an estimate of spurious counts, the question of what number to assign to $\Delta = \Delta'$ is somewhat troublesome, because probability computations based on Poisson distribution are very sensitive to $\bar{n}\Delta$ and the resolving time Δ_d as defined on Fig. 3 is not invariable. As can be readily observed by inspection of Fig. 3, the resolving time Δ_d for a fixed chamber-amplifier combination depends on the pulse height and shape and on the discriminator setting V_d. For a fixed discriminator setting the resolving time changes with pulse height and shape, and for given pulse height and shape the resolving time changes with discriminator setting. As shown in Figs. 3 and 4, the resolving time is assumed not to be affected by time delays within the discriminator the pulse of which is cut off at t_2 or earlier. Presumably for pulses that barely exceed the discriminator setting, the pulse emitted by the discriminator could extend beyond t_2 and extend the amplifier-discriminator resolving time beyond Δ_d; that is, when Δ_d as shown on Fig. 3 is small because pulse height barely exceeds V_d, the interval $t_2 - t_1$ becomes very small and time delays within the discriminator may become important. However, the problem of principal significance is the determination of the likelihood of pile-up of unwanted pulses within a resolving time $\Delta = \Delta_d$ sufficient to cause spurious counts when Δ_d is the resolving time that corresponds to the discriminator setting for average or representative pulses from the wanted radiation. This discriminator setting in turn depends on the frequency with which spurious counts may occur, and on the general noise level from other sources. If the pulse-height

discrimination could in effect be accomplished in the 10^{-6} volt region at the output of the chamber, the pile-up problem could be controlled there, and the RC time constants of the amplifier and the setting of the following discriminator could be adjusted to give the best signal-to-noise ratio from the point of view of neutron events and tube and thermal noises, but in general all of these circumstances are interwoven. Hence for use with a particular counting system, or for prediction in connection with a proposed counting system under design, a family of rates of requests should be computed using a range of $\Delta = \Delta_d$ in the vicinity of the resolving time Δ_d that is judged to be representative of system operation, which presumably is as shown in Figs. 3 and 4; or for design purposes the computations should be carried over a substantial range of Δ to survey the possibilities. By machine methods such computations can be carried over considerable ranges very readily and very rapidly.

The estimate of rate of spurious counts is computed from the estimate of the rate of requests from a knowledge or presumption of the manner in which the counting device responds to the requests. As indicated in Section I the counting device has a resolving time of its own which generally is at least several times the resolving time of the amplifier plus discriminator. Hence if the discriminator delivers requests to the counting device in too rapid succession, some of them are not recognized. Various modes of operation of the counting device may be conceived.

The dead time of the counting device may be imagined to be unaffected or unextended by pulses received from the discriminator during the resolving time Δ_e, so that another operation can begin as soon (practically immediately) after the termination of an operation as the counting device is re-energized either by continuation of the discriminator pulse or arrival of the next pulse. In other words, once energized, the counting device completes an operation in resolving time Δ_e regardless of additional pulses received or not received from the discriminator during the interim, and such operations continue consecutively until no discriminator pulse is present at the termination of an operation. Such operation, based on the unclipped discriminator pulses of Fig. 8*b* is, illustrated in Fig. 8*d*, and based on the clipped discriminator pulses of Fig. 8*c* is illustrated in Fig. 8*e*, for $\Delta_e/\Delta_d = 3$. The resistered count depends upon whether the individual operations are counted, for example 16 in Fig. 8*d* and 8 in Fig. 8*e*, or whether only the groups of consecutive operations are counted, 6 and 7, respectively. The analysis of these modes of operation is quite complicated, but the results can be summarized in readily usable form, as shown in Section IV. The supporting derivations are given in the full report (5), Appendices C, D and E.

Another mode of operation of the counting device, perhaps more realistic than the preceding scheme, is developed by imagining that the dead time is extended by discriminator pulses that are received during

the dead time, so that one operation cannot terminate until an interval at least as great as Δ_e occurs between discriminator pulses. In other words, the count registered then is equal to the number of intervals between discriminator pulses that equal or exceed Δ_e. Such operation, based on the unclipped discriminator pulses of Fig. 8*b*, is illustrated in Fig. 8*f*, and based on the clipped discriminator pulses of Fig. 8*c* is illustrated in Fig. 8g, likewise for $\Delta_e/\Delta_d = 3$. These illustrations indicate 2 and 7 counts, respectively. Analysis of these modes of operation is less complicated than analysis of the modes of operation illustrated by Fig. 8*d* and Fig. 8*e*; the results are summarized also in Section IV. The supporting derivations are given in the full report (5), Appendices C, D, and E.

Still other modes of operation of the counting device can be imagined and analyzed by methods analogous to the methods used to analyze the modes used herein as illustrations. For analysis of existing systems or design of new systems, entire families of results for counting rates can be readily computed for study by carrying $\bar{n}$, n_t, Δ_d and Δ_e over ranges of interest for various modes of system operation. These rates for spurious counts can be compared with the expected rates of neutron events to see whether or not the situations may be tolerable and as an aid to determining conditions required to make the situations tolerable. Theoretically a spurious pulse due to pile up of small unwanted pulses occasionally may coincide with a wanted pulse, so that the spurious pulse then may not give a count. It could, in fact, reduce the count by giving a resultant pulse sufficiently prolonged to overlap a succeeding wanted pulse, but since for satisfactory counting the frequency of spurious counts should be very low compared with the rate of desired counts. the chance of such coincidences should be very small. Likewise occasional overlaps of wanted pulses may occur and reduce the count. The probability of such pile-up can be analyzed by methods similar to the method used herein for alpha pile-up.

The general method is not limited to the study of alpha pile-up; it can be used for similar study of pile-up of pulses due to gamma radiation when the problem is to count pulses initiated by alpha particles in the presence of a background of gamma radiation. Also, if other than Poisson distribution applies, and the probable number of events that originate in a time interval Δ can be computed, the entire analysis in the same form as herein, can be derived on the basis of such probabilities.

III. RATES OF REQUEST—SUMMARY AND DISCUSSION OF RESULTS

When only black and white intervals exist, the probable number of *groups* of black intervals in a unit of time can be determined by examining the interval succeeding each black interval. If the interval is white, it must terminate a group of black intervals. Since the probable number of black intervals in a unit of time is $D_i = P_+/\Delta$, (which as discussed

in Section II would be the rate of requests if the discriminator could recognize individual black intervals even though consecutive), the probable number of times that a black interval is followed by a white interval is therefore

$$D_g = P_+P_-/\Delta = D_iP_-, \tag{8}$$

in a unit of time, which is the number of groups of black intervals in a unit of time, or the rate of requests in the absence of gray intervals. A plot of $D_g\Delta$ for a comparison with $D_i\Delta$ is shown on Fig. 6 for $\bar{n}\Delta = 10$. The $D_g\Delta$ curve approaches zero for high n_i just as the $D_i\Delta$ curve approaches zero, because P_+ approaches zero, but the $D_g\Delta$ curve theoretically has a maximum for

$$P_+ = P_- = 0.5 \tag{9}$$

or

$$P_+P_- = 0.25, \tag{10}$$

since

$$P_+ + P_- = 1, \tag{11}$$

and then approaches some smaller probability for $n_i = 0$ which for Fig. 6 is essentially zero, but for a different $\bar{n}\Delta$ may be relatively high, as shown on Fig. 9 for $\bar{n}\Delta = 1$. Since n_i must be integral, the parts of the curves of Figs. 6 and 9 that are between the plotted points really have no physical significance and the curves serve merely to give assistance in visualizing rates of requests as functions of discriminator settings. Hence unless Eq. 9 happens to be satisfied by an integral n_i, the maxima shown for D_g have no physical significance, but as $\bar{n}\Delta$ increases, the chance of the maximum falling near an integral n_i increases.

When in two adjacent time intervals Δ, Fig. 7, n and m events originate, respectively, with the restrictions that

$$1 \leq n \leq n_i, \tag{12}$$

$$1 \leq m \leq n_i, \tag{13}$$

and

$$(n_i + 1) \leq (n + m) \leq 2n_i, \tag{14}$$

the probability that more than n_i events originate in a time interval $\Delta' = \Delta$ that overlaps the adjacent time intervals Δ, Fig. 7, is

$$P'_{n+m} = \sum_{a=1}^{a=n+m-n_i} \left(\frac{1}{2}\right)^{n_i+a}, \tag{15}$$

which in other words is the probability that the two adjacent white intervals contain a gray interval. The $(1/2)^{n_i+a}$ appears because the two adjacent white intervals are equal and the distribution of events

within them is random. The probability that n events originate in a time interval Δ is P_n, Eq. 1, and the probability that m events originate in a time interval Δ is similarly P_m, so that the probability that a time interval Δ in which n events originate is followed by a time interval Δ

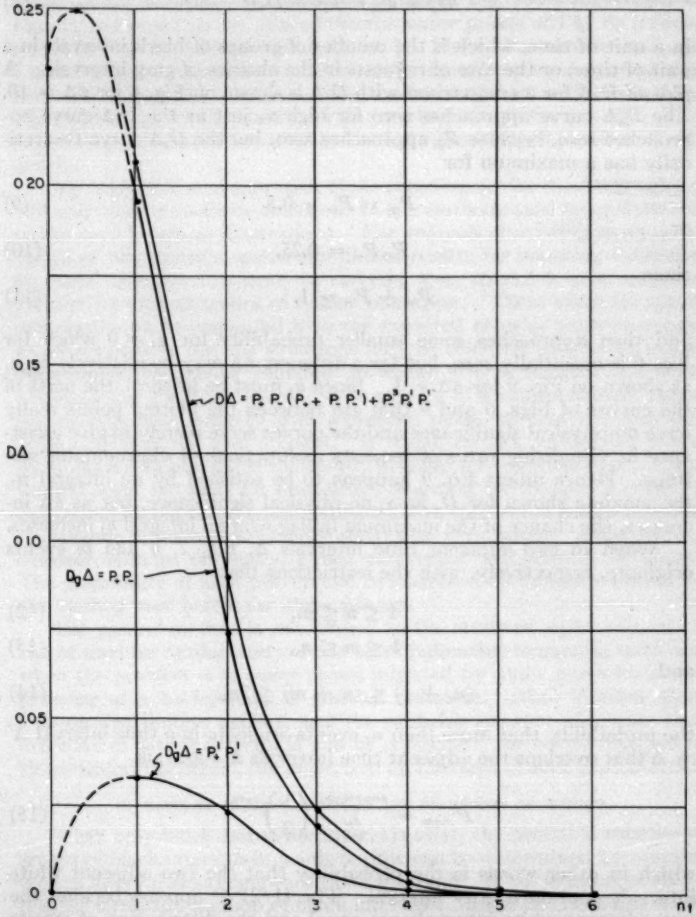


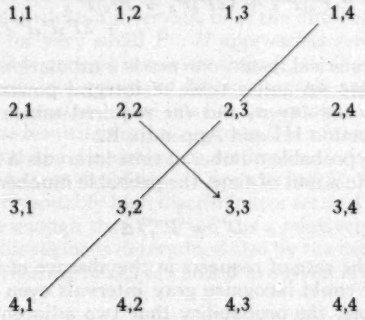
FIG. 9. Plots of $D\Delta$, $D_0\Delta$, and $D'_0\Delta$ for $n\Delta = 1$.

in which m events originate is P_nP_m, and the probability that these two adjacent intervals contain a $\Delta' = \Delta$ interval, Fig. 7, in which more than n_t events originate, in other words, a gray interval, is

$$P'_+ = P_nP_mP'_{n+m}. \tag{16}$$

The probabilities P_n and P_m, which give the chances that exactly n and m events originate respectively in adjacent intervals Δ, Fig. 7, are given by Eq. 1. If more than n_t events originate in either of the adjacent Δ intervals, an overlapping Δ' interval is not of interest; if no event originates within either of the adjacent Δ intervals, or if the sum of the number of events n and m that originate in the two adjacent intervals does not exceed n_t, more than n_t events cannot originate in an overlapping interval Δ'.

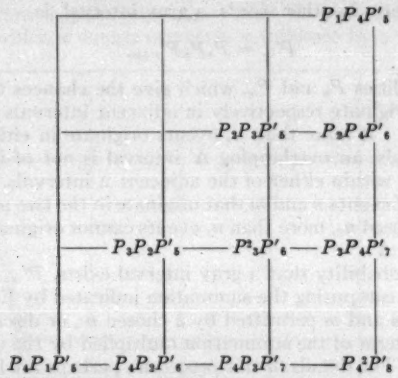
The total probability that a gray interval exists, P'_+, Eq. 16, must be evaluated by computing the summation indicated by Eq. 15 for each combination of n and m permitted by a chosen n_t, or discriminator setting, with each term of the summation multiplied by the corresponding product P_nP_m. The details of this procedure perhaps are best indicated by means of an array. First, for simplicity of illustration, specific numbers are used, for example, if n_t is 4, then n and m may have integral values 1 to 4 inclusive, and the possible permutations of these values, taken two at a time, are given in the following array:



In this array, the combinations above the diagonal are not permitted, because in each combination the sum is less than $n_t + 1$, in violation of Eq. 14. Hence the diagonal may be taken as indicating the discriminator setting: if n_t is 5, one more row is added on the bottom, and one more column is added on the right, and the diagonal moves accordingly, so that the numbers now on it then lie above it. In other words, as the discriminator setting is raised from the lowest setting, the diagonal may be visualized as sweeping from the 1,1 corner in the arrow direction, and

making a bigger and bigger square. In terms of Eq. 16 the array may be represented:

$P'_+ =$



(16a)

In this array, the subscripts on P are the values of n and m, and the subscripts on P' are the corresponding sums of n and m. The evaluation of the array is the summation of the terms,

$$P'_+ = 2(P_1P_4 + P_2P_3)P'_5 + (2P_2P_4 + P^2_3)P'_6 + 2P_3P_4P'_7 + P^2_4P'_8. \quad (16b)$$

To compute a numerical result, one needs a tabulation of Eq. 1 *versus* n for the appropriate $\bar{n}\Delta$ and a table of integral powers of (1/2). The generalized array for any n_i and the required tables are given in the full report (5), Section III and Appendix B.

Now since the probable number of time intervals Δ in which n events originate is P_n/Δ in a unit of time, the probable number of gray intervals in a unit of time is

$$D'_i = P'_+/\Delta \quad (17)$$

which would be the rate of requests in the absence of black intervals if the discriminator could recognize gray intervals even though consecutive. Furthermore, the probability that two adjacent white intervals do not contain a gray interval is

$$P'_- = 1 - P'_+, \quad (18)$$

so that by argument similar to the argument for the groups of black intervals, the probable number of times that a gray interval is followed by a white Δ' interval is

$$D'_g = P'_+P'_-/\Delta' = P'_+P'_-/\Delta = D'_iP'_- \quad (19)$$

in a unit of time, which is the number of gray groups in a unit of time, or the rate of requests in the absence of black intervals. A curve of $D'_g\Delta$ is shown in Fig. 9 in comparison with $D_g\Delta$ for $\bar{n}\Delta = 1$, because for $\bar{n}\Delta = 10$, Fig. 6, $D'_g\Delta$ is negligible. These curves are of different shape because they are different functions of n_t, but the $D'_g\Delta$ curve has a theoretical maximum also for

$$P'_+ = P'_- = 0.5, \tag{20}$$

or

$$P'_+P'_- = 0.25, \tag{21}$$

but practically this situation cannot be achieved physically, because P'_{+-} cannot exceed $(1/2)^2$ or 0.25, corresponding to $n_t = 1$, and at $n_t = 0$, $P'_+ = 0$. What happens to P'_+ between $n_t = 0$ and $n_t = 1$ is a meaningless question.

The resultant rate of requests is not in general $D_g + D'_g$, but is

$$D = D_b + D_x = [P_+P_-(P_+ + P_-P'_-) + P^2_-P'_+P'_-]/\Delta, \tag{22}$$

shown also in Fig. 9 in terms of $D\Delta$; D_b and D_x are the portions of D that originate with black or gray intervals, respectively. For very small P'_+, use of Eqs. 5 and 11 shows that D approaches D_g, and that for very small P_+, D approaches D'_g. Likewise, for very small P'_-, D approaches $P^2_+P_-/\Delta$, which is the number of black groups in a unit of time that are separated by single white Δ intervals, or is the number of isolated white Δ intervals, and for very small P_-, D approaches zero. The resultant rate of requests D in general is smaller than $D_g + D'_g$ because groups of gray intervals may join groups of black intervals and thus reduce the number of distinct groups. The relative importance of the gray groups in determining the resulting rate of requests D is a function of the density of events, $\bar{n}\Delta$, and the discriminator setting, n_t. As indicated by the limiting conditions just described, D approaches D'_g for low P_+, which generally means reasonably high discriminator setting, but in that region P'_+ may be large enough that $P^3_-P'_+P'_-$ is a relatively significant term. The location of this region is determined also by the density of events $\bar{n}\Delta$. Full derivations relating to D_g, D'_g and D are given in the full report (5), Appendix A.

IV. COUNTING RATES—SUMMARY AND DISCUSSION OF RESULTS

As indicated in the latter paragraphs of Section II and illustrated in Figs. 8*d* through 8*g*, the count given by the counting device depends on whether the output of the discriminator is unclipped or clipped, and how the dead time of the counting device is influenced by requests the counting device receives during the time interval Δ_c required for completion of an operation. When the output of the discriminator is unclipped, it

consists of a series of pulses of essentially the same constant height, Fig. 8*b*, the durations of which correspond essentially to the durations of the groups of black-gray pulses delivered to the discriminator by the pulse amplifier. When the output of the discriminator is clipped, it approaches a series of impulses each of the same height, Fig. 8*c*, that are initiated by the groups of black-gray pulses delivered to the discriminator by the pulse amplifier. For each of these types of discriminator output the occurrence of a request within a time interval less than Δ_e subsequent to the initiation of a counting operation by a preceding request, may or may not be imagined to interfere with the completion of the operation, as outlined in Section II. Hence four modes of counting operations are illustrated under these assumptions:

1. Discriminator output unclipped, counter operations completed in time Δ_e even though additional requests are made within Δ_e, and continuing consecutive operations if discriminator pulse continues beyond Δ_e: Fig. 8*d*.
2. Discriminator output clipped, counter operations completed in time Δ_e even though additional requests are made within Δ_e: Fig. 8*e*.
3. Discriminator output unclipped, counter operation once initiated not completed until an interval greater than Δ_e occurs between successive discriminator output pulses: Fig. 8*f*.
4. Discriminator output clipped, counter operations once initiated not completed until an interval greater than Δ_e occurs between successive requests: Fig. 8g.

The analyses to derive the counting rates for these four modes of counting operations involve determination of the distribution of x, x', z, and z', Figs. 8*b* and 8*c*, measured in terms of Δ, or Δ' and the use of certain auxiliary probabilities. The derivations are quite lengthy and involved; they are given in the full report (5), Appendices C, D, and E. The results are given in Tables I to VI and Figs. 10 to 13.

The conditions for the counting rate for Mode 1 have two facets: the counting rate may correspond to individual operations of the counting device even though consecutive, or it may correspond to groups of operations of the counting device. These two rates are directly related through

$$C^e{}_1 = [P^2{}_+P_- + P^2{}_-P'_-(1 + P_+)]C^i{}_1 \qquad (23)$$

in which $C^i{}_1$ and $C^e{}_1$ represent, respectively, the counting rates for individual operations and for groups of operations. Hence a knowledge of either $C^i{}_1$ or $C^e{}_1$ permits computation of the other rate through use of Eq. 23. Computation of $C^e{}_1$ is relatively easy, because the number of groups of operations of the counting device in a unit of time is equal to the number of spaces between discriminator output pulses, or the sum of the numbers of x and x' intervals, in a unit of time, that are not over-

lapped by operations of the counting device, as illustrated by comparison of Figs. 8*b* and 8*d*. This rate is equal to D, Eq. 22, if Δ_c does not exceed Δ_d in duration, or in other words, if the ratio of resolving times

$$\frac{\Delta_c}{\Delta_d} = r \leq 1. \tag{24}$$

For larger r, the rate is less than D, because some of the various x and x' intervals that contain fewer than r intervals Δ or Δ' may be overlapped by operations of the counting device.

The conditions for the counting rate for Mode 2 also have two facets, similar in kind to the two facets for Mode 1: the counting rate may correspond to individual operations of the counting device even though consecutive, or it may correspond to groups of operations of the counting device. The only circumstance under which operations of the counting device may be consecutive is for

$$r = kz, \tag{25}$$

or

$$r = kz', \tag{26}$$

in which k is an integer. All the clipped discriminator output pulses that occur during the operation of the counting device must be regarded as lost, whereas those clipped discriminator pulses spaced to satisfy Eq. 25 or Eq. 26 must be regarded as lost (except the first pulse in a row of them) in determining the counting rate for individual operations. For Mode 2 the two rates also are directly related, though not quite as simply as for Mode 1, through

$$C^g{}_{2r} = C^i{}_{2b}(1 - P_{bb}) + C^i{}_{2z}(1 - P_{zz}) \tag{27}$$

for r integral, and

$$C^g{}_{2h} = C^i{}_{2b}(1 - P_{zb}) + C^i{}_{2z}(1 - P_{bz}) \tag{28}$$

for r nonintegral. Here $C^i{}_{2b}$ and $C^i{}_{2z}$ represent, respectively, the counting rates for individual operations that are initiated by black and by gray pulses delivered to the discriminator by the pulse amplifier, and

$$C^i{}_2 = C^i{}_{2b} + C^i{}_{2z} = \frac{D_b}{1 + P_{c2}} + \frac{D_z}{1 + P'_{c2}} = D - C^i{}_{2b}P_{c2} - C^i{}_{2z}P'_{c2}, \tag{29}$$

the total counting rate for individual operations; $C^g{}_2$ represents the counting rate for groups of operations of the counting device, P_{bb} is the probability that an individual operation of the counting device initiated by a black pulse delivered to the discriminator has a like operation ad-

jacent to it, P_{aa} is the probability that an individual operation of the counting device initiated by a gray pulse delivered to the discriminator has a like operation adjacent to it, P_{ab} is the probability that an individual operation of the counting device initiated by a gray pulse delivered to the discriminator is adjacent to an operation initiated by a black pulse delivered to the discriminator, and P_{ba} is the probability that an individual operation of the counting device initiated by a black pulse delivered to the discriminator is adjacent to an operation initiated by a gray pulse delivered to the discriminator; P_{c2} is the probability that a discriminator pulse will originate within Δ_c when the counting operation is initiated by a discriminator pulse that originates in a black interval, and P'_{c2} is the probability that a discriminator pulse will originate within Δ_c when the counting operation is initiated by a discriminator pulse that originates in a gray interval. Evidently a knowledge of C'_{2b} and C'_{2a} as components of C'_2 is necessary for computation of C''_2 from C'_2 or vice versa. The number of individual operations of the counting device in a unit of time is equal to the number of clipped discriminator pulses in a unit of time reduced by the number of such pulses in a unit of time that occur during the operation of the counting device, as illustrated by comparison of Figs. 8*c* and 8*e*. This rate is equal to D, Eq. 22, if Δ_c does not exceed $2\Delta_d$, or

$$r \leq 2, \tag{30}$$

because the minimum possible spacing of the clipped discriminator pulses, according to the assumptions the method involves, is $2\Delta_d$. For larger r, the rate is less than D, and the reductions must be made in terms of C'_{2b} and C'_{2a} separately, based on the distribution of z and z' in terms of Δ or Δ' and the probable sequences of z and z' intervals.

The counting rate for Mode 3 is obtained very simply. The counting device once an operation is started, remains paralyzed until an x or x' interval, Figs. 8*b* and 8*c*, longer than Δ_c occurs, as illustrated by comparison of Figs. 8*b* and 8*f*. Since the rate at which the sum of x and x' intervals occurs is D, Eq. 22, the counting rate for Mode 3 is equal to the rate of requests, D, reduced by the rate of occurrence of x and x' intervals that do not exceed Δ_c, or

$$C_3 = D - \sum_{1}^{r} (X + X') \tag{31}$$

in which X and X' are the respective rates at which

$$x = w\Delta \text{ or } h\Delta \tag{32}$$

and

$$x' = w\Delta' \text{ or } h\Delta' \tag{33}$$

occur, as shown in Table I.

TABLE I.

x or x'	$X+X'$, number/unit time
Δ	$[P^2{}_+P_-+P_-(P_-P'_+)^2P_-P'_-]/\Delta=(X+X')_1$
2Δ	$P_-P'_-(X+X')_1$
3Δ	$(P_-P'_-)^2(X+X')_1$
.	.....
.	.....
$w\Delta$	$(P_-P'_-)^{w-1}(X+X')_1$
1.5Δ	$2P_+P_-(P_-P'_+)(P_-P'_-)/\Delta=(X+X')_{1.5}$
2.5Δ	$P_-P'_-(X+X')_{1.5}$
3.5Δ	$(P_-P'_-)^2(X+X')_{1.5}$
.	.....
.	.....
$h\Delta$	$(P_-P'_-)^{h-1.5}(X+X')_{1.5}$

Likewise the counting rate for Mode 4 is obtained very simply. The counting device, once an operation is started, remains paralyzed until a z or z' interval, Figs. 8*b* and 8*c*, longer than Δ_c occurs, as illustrated by comparison of Figs. 8*c* and 8*g*. The rate at which the sum of z and z' intervals occurs also is D, Eq. 22 so that the counting rate for Mode 4 is equal to the rate of requests, D, reduced by the rate of occurrence of z and z' intervals that do not exceed Δ_c, or

$$C_4 = D - \sum_2^r (Z + Z') \tag{34}$$

in which Z and Z' are the respective rates at which

$$z = w\Delta \text{ or } h\Delta \tag{35}$$

and

$$z' = w'\Delta' \text{ or } h'\Delta' \tag{36}$$

occur, as shown in Tables II and III. Since adjacent clipped discriminator pulses cannot be separated by less than 2Δ or $2\Delta_c$, z and z' cannot be less than 2Δ or $2\Delta'$, and reduction of the counting rate below D therefore requires

$$r \geq 2. \tag{37}$$

With $X + X'$ available, Table IV can be used to obtain $C^e{}_1$, and Eq. 23 then can be used to obtain $C^i{}_1$. With Z and Z' available, Tables V and VI can be used to obtain P_{e2}, P'_{e2}, P_{bb}, P_{zz}, P_{zb} and P_{bz}; successive use of Eqs. 29, 28, and 27 then gives $C^i{}_2$ and $C^e{}_2$.

The entire analysis for counting rates has been made in terms of

steps of 0.5 in r, and in general the solution that applies at one step continues to apply until the next step is reached. Actually, owing to the manner in which half intervals have been defined, and to the fact that Δ_d itself is not absolutely fixed in a physical situation, these transitions probably are not sharp. Ordinarily Δ_e is large compared with Δ_d, so

TABLE II.

z	Z, number/unit time
2Δ	$(P_+P_-)^2(P_++P_-P'_-)/\Delta = Z_1$
3Δ	$(P_+P_-)^3(P_++P_-P'_-)^2/\Delta = Z_2$
4Δ	$Z_3-(P_+P_-)Z_2$
5Δ	$Z_4-(P_+P_-)(P_++P_-P'_-)Z_3-(P_+P_-P'_-)Z_{2.5}-(P_+P_-)Z_3$
6Δ	$Z_5-(P_+P_-)(P_++P_-P'_-)(Z_3+Z_{2.5}+Z_3)-(P_+P_-P'_-)Z_{3.5}-(P_+P_-)Z_4$
.	.....
.	.....
$w\Delta$	$Z_{w-1}-(P_+P_-)(P_++P_-P'_-)(Z_3+Z_{3.5}+Z_4+\cdots+Z_{w-3.5}+Z_{w-3})-(P_+P_-P'_-)Z_{w-3.5}-(P_+P_-)Z_{w-2}$
2.5Δ	$(P_+P_-)(P_++P_-P'_-)(P^2_-P'_++P'^2_-)/\Delta = Z_{2.5}$
3.5Δ	$(P_+P_-)(P_++P_-P'_-)(P^2_-P'_++P'^2_-)/\Delta = Z_{3.5} = Z_{2.5}$
4.5Δ	$Z_{3.5}-(P^2_-P'_+P'^2_-)Z_3-(P^2_-P'_+P'^2_-)Z_{2.5}$
5.5Δ	$Z_{4.5}-(P^2_-P'_+P'^2_-)(Z_3+Z_{2.5}+Z_3)-(P^2_-P'_+P'^2_-)Z_{3.5}$
6.5Δ	$Z_{5.5}-(P^2_-P'_+P'^2_-)(Z_3+Z_{3.5}+Z_3+Z_{3.5}+Z_4)-(P^2_-P'_+P'^2_-)Z_{4.5}$
.	.....
.	.....
$h\Delta$	$Z_{h-0.5}-(P^2_-P'_+P'^2_-)Z_3+Z_{3.5}+Z_3+Z_{3.5}+Z_4+\cdots+Z_{h-3}+Z_{h-2.5})-(P^2_-P'_+P'^2_-)Z_{h-2}$

TABLE III.

z'	Z', number/unit time
$2\Delta'$	$(P^2_-P'_+P'^2_-)(P^2_-P'_+P'^2_-)/\Delta = Z'_2$
$3\Delta'$	$(P^2_-P'_+P'^2_-)^2/\Delta = Z'_3$
$4\Delta'$	$Z'_3-(P^2_-P'_+P'^2_-)Z'_2$
$5\Delta'$	$Z'_3-(P^2_-P'_+P'^2_-)(Z'_2+Z'_{2.5})-(P^2_-P'_+P'^2_-)Z'_3$
$6\Delta'$	$Z'_3-(P^2_-P'_+P'^2_-)(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5})-(P^2_-P'_+P'^2_-)Z'_4$
.	.....
.	.....
$w'\Delta'$	$Z'_3-(P^2_-P'_+P'^2_-)(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5}+\cdots+Z'_{w'-3}+Z'_{w'-2.5})-(P^2_-P'_+P'^2_-)Z'_{w'-2}$
$2.5\Delta'$	$(P^2_-P'_+P'^2_-)(P_+P_-P'_-)/\Delta = Z'_{2.5}$
$3.5\Delta'$	$(P^2_-P'_+P'^2_-)(P_+P_-)(P_++P_-P'_-)/\Delta = Z'_{3.5}$
$4.5\Delta'$	$Z'_{3.5}-(P_+P_-P'_-)Z'_2-(P_+P_-)Z'_{2.5}$
$5.5\Delta'$	$Z'_{3.5}-(P_+P_-)(P_++P_-P'_-)(Z'_2+Z'_{2.5})-(P_+P_-P'_-)Z'_3-(P_+P_-)Z'_{3.5}$
$6.5\Delta'$	$Z'_{3.5}-(P_+P_-)(P_++P_-P'_-)(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5})-(P_+P_-P'_-)Z'_4-(P_+P_-)Z'_{4.5}$
.	.....
.	.....
$h'\Delta'$	$Z'_{3.5}-(P_+P_-)(P_++P_-P'_-)(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5}+\cdots+Z'_{h'-3.5}+Z'_{h'-3}$ $-(P_+P_-P'_-)Z'_{h'-2.5}-(P_+P_-)Z'_{h'-2}$

that r is large compared with 0.5, and differences in counting rate for adjacent steps in r should not be great. For small r, the differences of course could be considerable, but in any case the differences depend also upon the general proportions of the situation: the average rate of events, the discriminator setting, and the resolving time of the amplifier-discriminator, that determine the various probabilities involved.

As illustrations, counting rates C_3 and C_4 are computed for $\bar{n}\Delta = 10$ and for $\bar{n}\Delta = 1$, and are compared with D_i (or $D_i + D'_i$) and D (or

TABLE IV.

r	C_{r_1}, Group Counting Rate
1	D
1.5	$\dfrac{D-(X+X')_1}{1-[P^2_+ + (P_-P'_+)^2 P_-]} = \dfrac{D-(X+X')_1}{1-Q_1}$
2	$\dfrac{D-(X+X')_1-(X+X')_{1.5}}{1-Q_1-2P_+P^2_-P'_+} = \dfrac{D-(X+X')_1-(X+X')_{1.5}}{1-Q_1-Q_{1.5}}$
2.5	$\dfrac{D-(X+X')_1-(X+X')_{1.5}-(X+X')_2}{1-Q_1-Q_{1.5}-Q_1P_-P'_-} = \dfrac{D-(X+X')_1-(X+X')_{1.5}-(X+X')_2}{1-Q_1(1+P_-P'_-)-Q_{1.5}}$
3	$\dfrac{D-(X+X')_1-(X+X')_{1.5}-(X+X')_2-(X+X')_{2.5}}{1-Q_1(1+P_-P'_-)-Q_{1.5}-Q_{1.5}P_-P'_-} = \dfrac{D-\sum_1^{r-0.5}(X+X')}{1-(Q_1+Q_{1.5})(1+P_-P'_-)}$
3.5	$\dfrac{D-\sum_1^{r-0.5}(X+X')}{1-Q_1[1+P_-P'_-+(P_-P'_-)^2]-Q_{1.5}(1+P_-P'_-)}$
4	$\dfrac{D-\sum_1^{r-0.5}(X+X')}{1-(Q_1+Q_{1.5})[1+P_-P'_-+(P_-P'_-)^2]}$
r_h*	$\dfrac{D-\sum_1^{r-0.5}(X-X')}{1-Q_1[1+P_-P'_-+(P_-P'_-)^2+\cdots+(P_-P'_-)^{r-1.5}]-Q_{1.5}[1+P_-P'_-+(P_-P'_-)^2+\cdots+(P_-P'_-)^{r-2.5}]}$
r_∞*	$\dfrac{D-\sum_1^{r-0.5}(X+X')}{1-(Q_1+Q_{1.5})[1+P_-P'_-+(P_-P'_-)^2+\cdots+(P_-P'_-)^{r-2}]}$

* r_∞ indicates the general result for r integral,
r_h indicates the general result for r nonintegral.

D_g), Figs. 10, 11, 12, and 13. To accomplish the computation of C_3 and C_4, the values of P_+, P_-, P'_+, and P'_- already used for computation of D, Figs. 6 and 9 are used to compute $X + X'$ and $Z + Z'$, as indicated in Tables I, II, and III, for use in Eqs. 31 and 34.

On Figs. 10 and 11, for $\bar{n}\Delta = 10$, the top curve, marked $D_i\Delta$, taken from Fig. 6, represents the rate of requests only if the amplifier-discriminator combination can recognize individually each Δ interval in which more than n_i events originate, and represents the counting rate only if the counting device can recognize each request. The next curve,

marked $D\Delta$, also taken from Fig. 6, represents the actual rate of requests, reduced below $D_i\Delta$ because the amplifier-discriminator combination cannot recognize separately consecutive intervals in which more than n_i events originate, and represents the counting rate only if the counting device can recognize each request. The remaining curves show the reduction in counting rate when the counting device cannot recognize all the requests individually. For $\bar{n}\Delta = 10$ the contributions of the Δ' intervals are negligible, which explains the neglect of D'_i in the top curve, and the substantial equivalence $D \cong D_s$.

TABLE V.—*Probabilities P_{c2} and P'_{c2}.*

r	P_{c2}
1	0
1.5	0
2	0
2.5	Z_2/D_b
3	$(Z_2+Z_{2.5})/D_b$
3.5	$(Z_2+Z_{2.5}+Z_3)/D_b$
4	$(Z_2+Z_{2.5}+Z_3+Z_{3.5})/D_b$
4.5	$(Z_2+Z_{2.5}+Z_3+Z_{3.5}+Z_4)/D_b+(Z_2/D_b)^2$
5	$(Z_2+Z_{2.5}+Z_3+Z_{3.5}+Z_4+Z_{4.5})/D_b+(Z^2_2+Z_2Z_{2.5})D^2_b+Z_{2.5}Z'_2/D_bD_s$
5.5	$(Z_2+Z_{2.5}+Z_3+Z_{3.5}+Z_4+Z_{4.5}+Z_5)/D_b+(Z_2Z_{2.5}+Z_2Z_3+Z_2Z_4)/D^2_b+(Z_{2.5}Z'_2+Z_{2.5}Z'_{2.5})D_bD_s$

r	P'_{c2}
1	0
1.5	0
2	0
2.5	Z'_2/D_s
3	$(Z'_2+Z'_{2.5})/D_s$
3.5	$(Z'_2+Z'_{2.5}+Z'_3)/D_s$
4	$(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5})/D_s$
4.5	$(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5}+Z'_4)/D_s+(Z'_2/D_s)^2$
5	$(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5}+Z'_4+Z'_{4.5})/D_s+[(Z'_2)^2+Z'_2Z'_{2.5})]D^2_s+Z'_{2.5}Z_2/D_sD_b$
5.5	$(Z'_2+Z'_{2.5}+Z'_3+Z'_{3.5}+Z'_4+Z'_{4.5}+Z'_5)/D_s+(Z'_2Z'_{2.5}+Z'_2Z'_3+Z'_2Z'_4)/D^2_s$ $+(Z'_{2.5}Z_2+Z'_{2.5}Z_{2.5})/D_sD_b$

On Figs. 12 and 13, for $\bar{n}\Delta = 1$, the contribution of the Δ' intervals is substantial, and has been included in the top curve, and in the $D\Delta$ curve taken from Fig. 9, the interpretations of which are the same as the interpretations of the corresponding curves of Figs. 10 and 11 for $\bar{n}\Delta = 10$. The remaining curves on Figs. 12 and 13 show the reduction in counting rate when the counting device cannot recognize all the requests individually.

Owing to change of scale, the crowding and reduction in counting rate in Figs. 10 and 11 (which show considerably greater ranges of D_i and n_i than Figs. 12 and 13) seem much greater than the crowding and

reduction in Figs. 12 and 13 caused by inability of the counting device to discriminate. Actually the ordinates of Figs. 10 and 11 must be multiplied by 4, and the abscissas must be multiplied by 2 to make those plots directly comparable with the plots of Figs. 12 and 13. The reduction is greater for C_3 (discriminator output unclipped) than for C_4 (disciminator output clipped), as should be expected, because when the discriminator pulses are clipped, a longer Δ_c is required to merge two requests than when pulses are not clipped.

The portions of the curves of Figs. 10 to 13 between plotted points are meaningless. Strictly, the curves probably should be drawn broken

TABLE VI.—*Probabilities* P_{bb}, P_{zz}, P_{zb}, *and* P_{bz}.

r	P_{bb} and P_{zz}
2	Z_2/D_b
	Z'_2/D_z
3	Z_3/D_b
	Z'_3/D_z
4	$Z_4/D_b + (Z_2/D_b)^2$
	$Z'_4/D_z + (Z'_2/D_z)^2$
5	$Z_5/D_b + (Z_2Z_3 + Z_3Z_2)/D_b^2 + Z_{2.5}Z'_{2.5}/D_bD_z$
	$Z'_5/D_z + (Z'_2Z'_3 + Z'_3Z'_2)/D_z^2 + Z'_{2.5}Z_{2.5}/D_zD_b$
r	P_{zb} and P_{bz}
2.5	$Z'_{2.5}/D_z$
	$Z_{2.5}/D_b$
3.5	$Z'_{2.5}/D_z$
	$Z_{2.5}/D_b$
4.5	$Z'_{4.5}/D_z + Z'_2Z'_{2.5}/D_z^2 + Z'_{2.5}Z_2/D_zD_b$
	$Z_{4.5}/D_b + Z_2Z_{2.5}/D_b^2 + Z_{2.5}Z'_2/D_bD_z$
5.5	$Z'_{5.5}/D_z + Z'_3Z'_{2.5}/D_z^2 + (Z'_{2.5}Z_3 + Z'_{3.5}Z_2)/D_zD_b + Z'_2Z'_{2.5}D_z$
	$Z_{5.5}/D_b + Z_2Z_{2.5}/D_z^2 + (Z_{2.5}Z'_3 + Z_{3.5}Z'_2)D_bD_z + Z_2Z_{2.5}/D_b$

line, because n_i cannot be fractional, but smoothing of the curves helps somewhat in visualizing the trends. However, when discriminator setting actually is made, it is made by voltage, and really corresponds to some kind of average n_i, owing to differences in chamber pulse heights and durations, and hence might be envisioned practically as corresponding to an intermediate point.

Similar curves readily can be computed and plotted for C^e_1, C^i_1, C^e_2, and C^i_2, though the arithmetic is somewhat more arduous than for C_3 and C_4 and the conditions perhaps are less realistic. Once programmed,

extensive families of counting rates can be obtained very quickly by use of a computer. Just what combination of n_t, Δ_e, and Δ_d is advantageous in a particular situation of course depends on what happens to the count of wanted pulses as well as on the reduction of count of unwanted pulses.

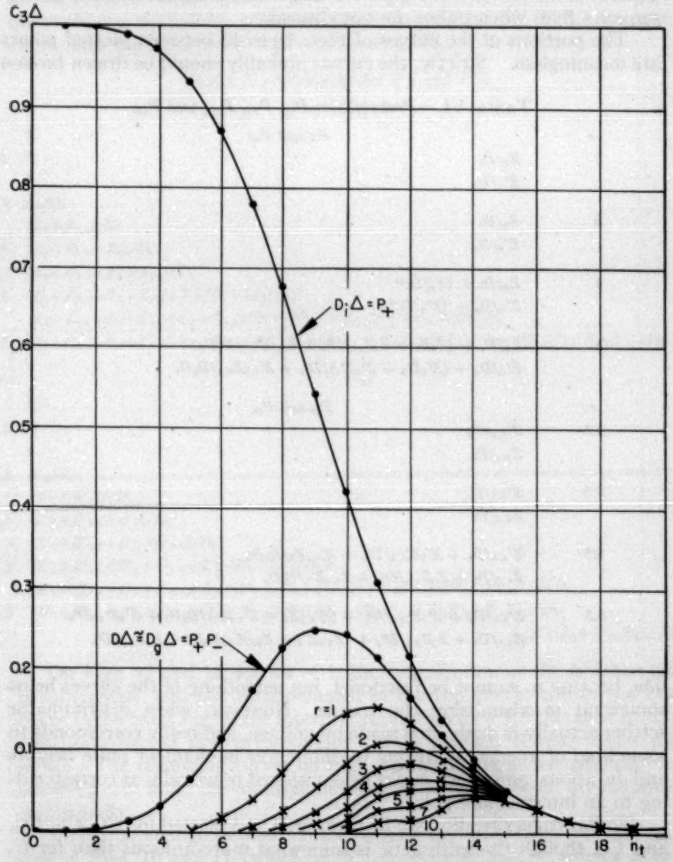


FIG. 10. Plots of $D_1\Delta$, $D\Delta$ and $C_3\Delta$ for $n\Delta = 10$.

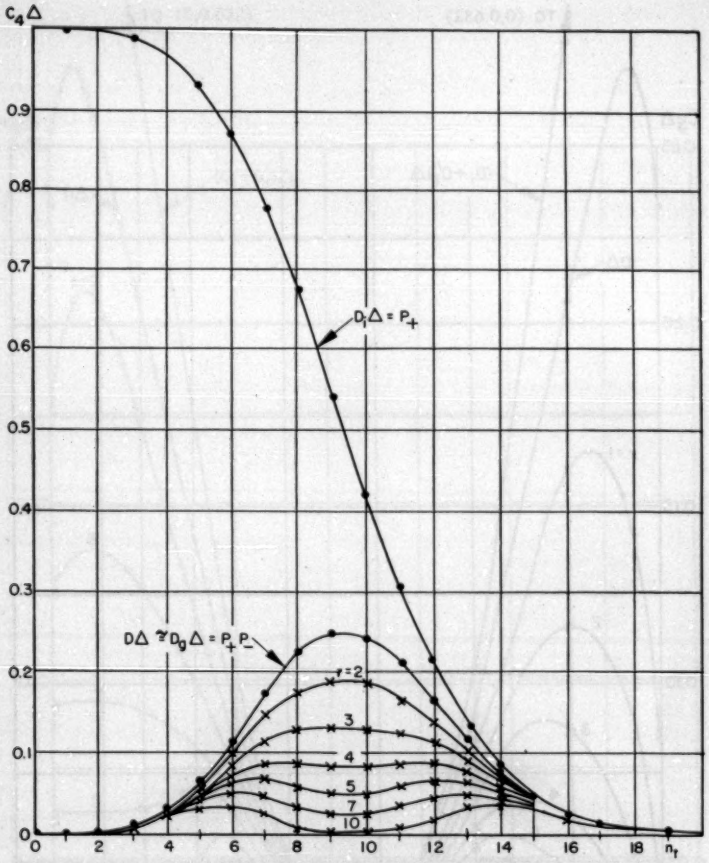


FIG. 11. Plots of $D_i\Delta$, $D\Delta$ and $C_4\Delta$ for $\bar{n}\Delta = 10$.

V. RESUMÉ

The probable rate at which pile-up beyond n_t events occurs in individual Δ or Δ' intervals is

$$D_i + D'_i = (P_i + P'_+)/\Delta \tag{38}$$

but owing to the fact that the discriminator does not recognize these individual black and gray intervals but recognizes only distinct groups

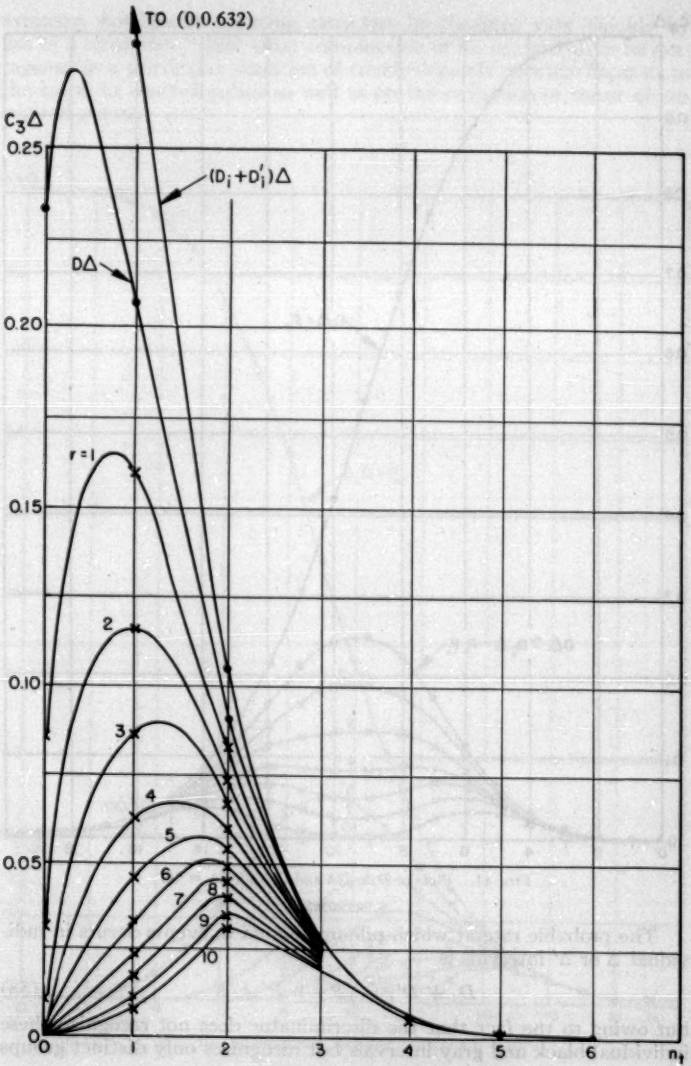


FIG. 12. Plots of $(D_i + D'_i)\Delta$, $D\Delta$, and $C_3\Delta$ for $n\Delta = 1$.

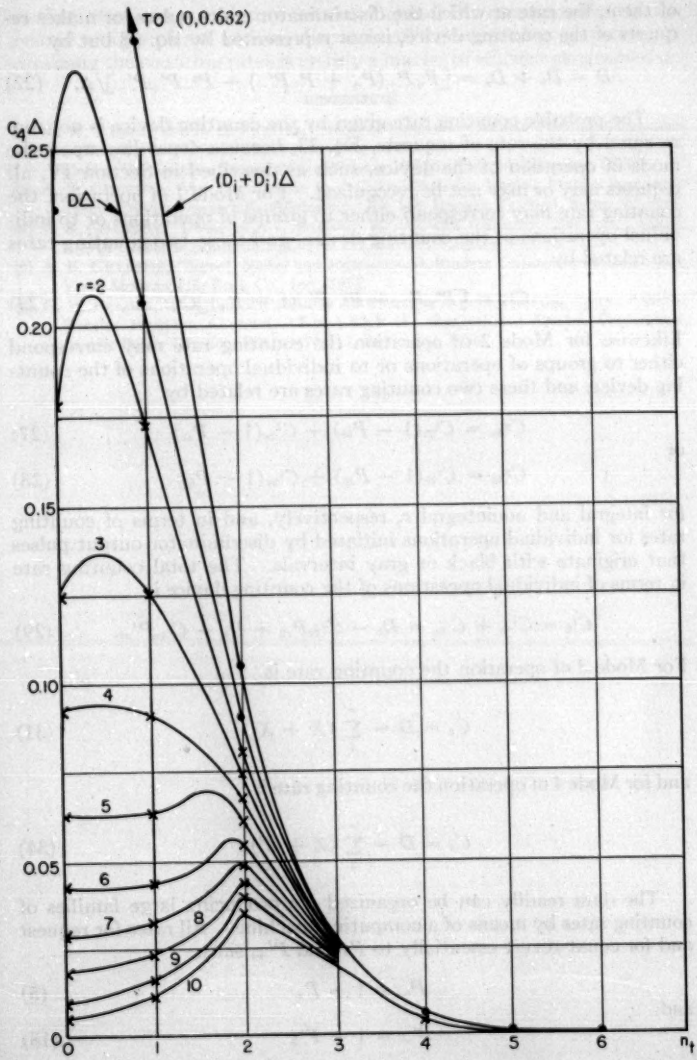


FIG. 13. Plots of $(D_i + D_i'\Delta)$, $D\Delta$, and $C_4\Delta$ for $\mathfrak{n}\Delta = 1$.

of them, the rate at which the discriminator emits pulses, or makes requests of the counting device, is not represented by Eq. 38 but by

$$D = D_b + D_x = [P_+P_-(P_+ + P_-P'_-) + P^2_-P'_+P'_-]/\Delta. \quad (22)$$

The probable counting rate given by the counting device is not represented by the rate of requests, Eq. 22, because depending upon the mode of operation of the device, such as described in Section IV, all requests may or may not be recognized. For Mode 1 of operation, the counting rate may correspond either to groups of operations or to individual operations of the counting device, and these two counting rates are related by

$$C^g_1 = [P^2_+P_- + P^2_-P'_-(1 + P_+)]C^i_1. \quad (23)$$

Likewise for Mode 2 of operation the counting rate may correspond either to groups of operations or to individual operations of the counting device, and these two counting rates are related by

$$C^g_{2w} = C^i_{2b}(1 - P_{bb}) + C^i_{2x}(1 - P_{xx}) \quad (27)$$

or

$$C^g_{2A} = C^i_{2b}(1 - P_{xb}) + C^i_{2x}(1 - P_{bx}) \quad (28)$$

for integral and nonintegral r, respectively, and in terms of counting rates for individual operations initiated by discriminator output pulses that originate with black or gray intervals. The total counting rate in terms of individual operations of the counting device is

$$C^i_2 = C^i_{2b} + C^i_{2x} = D_b - C^i_{2b}P_{e2} + D_x - C^i_{2x}P'_{e2}. \quad (29)$$

For Mode 3 of operation the counting rate is

$$C_3 = D - \sum_1^r (X + X') \quad (31)$$

and for Mode 4 of operation the counting rate is

$$C_4 = D - \sum_2^r (Z + Z'). \quad (34)$$

The data readily can be organized for producing large families of counting rates by means of a computing machine. All rates for request and for count revert essentially to P_+ and P'_+, since

$$P_- = 1 - P_+ \quad (5)$$

and

$$P'_- = 1 - P'_+ \quad (18)$$

so that after computation of sets of P_+ and P'_+ corresponding to the desired ranges of $\bar{n}\Delta$ and n_i, dispatch of the remaining operations in obtaining the counting rates is merely a matter of efficient programming.

REFERENCES

(1) W. W. RAMAGE AND R. H. FRAZIER, "Boost of Ionization Chamber Pulse by Subdivision or Transformer," Naval Reactor Program, Contract AT-11-1-GEN-14, Westinghouse Electric Corporation, Atomic Power Division, WAPD-RM-206, December, 1953.

(2) A. B. GILLESPIE, "An Introduction to A-C Amplifiers for Nuclear Physics Research," Atomic Energy Research Establishment, G/R 118, Harwell, England, August 15, 1947.

(3) A. B. GILLESPIE, "Design of A-C Amplifiers for Use with Ionization Chambers," Atomic Energy Research Establishment, G/R 168, Harwell, England, Feb. 24, 1948.

(4) A. B. GILLESPIE, "Signal, Noise, and Resolution in Nuclear Counter Amplifiers," New York, McGraw-Hill Book Co., Inc., 1953.

(5) R. H. FRAZIER, "Analysis of Pulse Pile-up Effects in a Pulse-Counting System," Naval Reactor Program, Contract AT-11-1-GEN-14, Westinghouse Electric Corporation, Atomic Power Division, WAPD-TM-13, September, 1956.

Raindrop Spectrometer.—University of Michigan meteorologists are particularly pleased with the rain that has fallen this year—they've invented a device that measures the exact size of raindrops, and are trying it out in each storm. The instrument is called a "raindrop spectrometer," and it can record the sizes of drops above one hundredth of an inch in diameter.

Its inventor, A. Nelson Dingle, associate research meteorologist in the University's Engineering Research Institute, says the device will mass produce much needed information on the sizes of the drops which make up our rains.

"Different rains have different character," he observes. "They vary in the amount of energy they have for eroding or compacting the soil, in their efficiency for washing dust particles—including radioactive debris—out of the atmosphere, and in the way they reflect radar waves back to observation stations."

The last point is of special interest to those who use radar to observe rainfall distribution, Dingle says. If the radar echoes from storms of different drop sizes can be catalogued, then meteorologists can measure by radar the rains that fall within the range of their transmitter.

"Instruments such as this may give rise to a whole new phase of climatology," he adds. "Raindrops can be measured above and below cloud formations, at different temperatures, in different geographical areas, and from different types of clouds." The device also has potential for measuring the effectiveness of man's "rain-making" operations, he thinks.

The instrument is set up on top of the U-M's East Engineering Building. It consists mainly of two black boxes mounted at the ends of two arms. One box contains a light source, the other a photoelectric cell. As the arms whirl around three times every second, the photocell "watches" a spot in the light beam. When a raindrop passes through that spot, the amount of light seen by the photocell tells the size of the drop. Because it is spinning so rapidly, the device scans a path about six yards around every second, thereby observing far more drops than it would if the arms stood still. In the heaviest rain, the instrument can record about 50 drops a second.

The unit appears deceptively simple. Inside a nearby laboratory, however, are located panels of electronic instruments needed to make the measurements and record data. After a test, dials show how many drops of each size fell in the sampling area. By measuring as many as 180,000 of them an hour, the U-M researchers expect to get a valid idea as to the distribution of drop sizes.

It takes a million average cloud droplets to form the average raindrop, Dingle points out, and meteorologists are interested in how nature combines millions of billions of droplets into billions of raindrops in a single storm.

In developing the instrument under a grant from the Air Force's Cambridge Research Center through the Engineering Research Institute, Dingle and his associates devised a slender "hyperdermic needle" which dripped drops of the exact size required. This was used to test and to calibrate the spectrometer.

With some refinements, the spectrometer should be able to measure drops as small as two-thousandths of an inch in diameter, Dingle thinks.

THE FRANKLIN INSTITUTE

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 472 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

AERONAUTICS

Van Sickle, Neil D. Modern Airmanship. 1957.

ASTRONOMY

Kuhn, Thomas S. The Copernican Revolution. 1957.
Wolley, R. van der Riet. A key to the Stars. Ed. 3. 1957.

BIBLIOGRAPHY

American Insititue of Electrical Engineers. Bibliography on High-Frequency Dielectric Heating. 1947.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Clarke, Hans Thacher, ed. Ion Transport Across Membranes. 1953.
Erickson, John G.; Wiley, Paul F. and Wystrach, V. P. The 1, 2, 3- and 1, 2, 4-Triazines, Tetrazines and Pentazines. 1956.
Fieser, Louis F. and Fieser, Mary. Introduction to Organic Chemistry. 1957.
Gaudin, A. M. Flotation. Ed. 2. 1957.
Rase, Howard F. and Barrow, M. H. Project Engineering of Process Plants. 1957.
Rochow, Eugene George; Hurd, Dallas T. and Lewis, Richard N. The Chemistry of Organo-Metallic Compounds. 1957.
Szmant, H. Harry. Organic Chemistry. 1957.
Vehrenkamp, Gerhard. Starkstrom Installationstechnik. Ed. 3. 1957.

CIVIL ENGINEERING

Pugsley, Alfred. The Theory of Supension Bridges. 1957.

COAL

Van Krevelen, D. W. and Schuyer, J. Coal Science. 1957.

CRYSTALLOGRAPHY

Guinier, Andre. Théorie et Technique de la Radiocristallographie. Ed. 2. 1956.

ELECTRICITY AND ELECTRICAL ENGINEERING

Dummer, G. W. A. Variable Capacitors and Trimmers. 1957.
Fisher, D. H. VHF Television Tuners. 1957.
Joint Electron Tube Engineering Council. Summary of Registered Transistors. 1956.
Jones, J. D. Frequency Modulation Receivers. 1957.
Koppelmann, F. Wechselstrommesstechnik. 1956.
Kosack, Hans-Joachim and Wangerin, Albert. Elektrotechnik auf Handelsschiffen. 1956.

McKENZIE, A. E. E. A Second Course of Electricity. 1957.
SHEA, RICHARD F., ED. Transistor Circuit Engineering. 1957.
WEBER, GEORG. Orienterende Elektroteknik. 1955.
WIND, MOE, ED. Handbook of Electronic Measurements. 2 Vol. 1956.
ZAREW, B. M. Berechnung und Konstruktion von Elektronenröhren. 1955.

GEOLOGY

FLINT, RICHARD FOSTER. Glacial and Pleistocene Geology. 1957.

HYDRAULIC ENGINEERING

JAEGER, CHARLES. Engineering Fluid Mechanics. 1957.

INDUSTRIAL MANAGEMENT

AMRINE, HAROLD T.; RITCHEY, JOHN A. AND HULLEY, OLIVER S. Manufacturing Organization and Management. 1957.

MANUFACTURE

BEGEMAN, MYRON LOUIS. Manufacturing Processes. Ed. 4. 1957.

MATHEMATICS

BUCKINGHAM, R. A. Numerical Methods. 1957.
JOHNSON, RICHARD E. AND KIOKEMEISTER, FRED L. Calculus with Analytic Geometry. 1957.
LUZADDER, WARREN JACOB. Graphics for Engineers. 1957.
THRALL, ROBERT McDOWELL AND TORNHEIM, LEONARD. Vector Spaces and Matrices. 1957.
WITTGENSTEIN, LUDWIG. Remarks on the Foundations of Mathematics. 1956.

OPTICS

LINSCHOTEN, JOHANNES. Strukturanalyse der Binokularen Tiefenwahrnehmung. 1956.
SCHILD, EWALD. Praktische Mikroskopie. Ed. 3. 1955.

PHOTOGRAPHY

ANGERER, LORENZ LUDWIG MAX ERNST VON. Wissenschaftliche Photographie. Ed. 6. 1956.

PHYSICS

BERTHELOT, ANDRÉ. Rayonnements de Particules Atomiques, Electrons et Photons. 1956.
CORK, JAMES M. Radioactivity and Nuclear Physics. Ed. 3. 1957.
DEKKER, ADRIANUS J. Solid State Physics. 1957.
JAMMER, MAX. Concepts of Force. 1957.
KEITZ, H. A. E. Light Calculations and Measurements. 1955.
RICCATI, VINCENZO. De' Principi della Meccanica. 1772.
ROCCHI, ANTONIO. Della Misura delle Velocità. 1773.
SCHEIDEGGER, ADRIAN E. The Physics of Flow Through Porous Media. 1957.
SHELDON, H. HORTON. Space, Time and Relativity. 1932.
VAN DE HULST, H. C. Light Scattering by Small Particles. 1957.
WORSTER, BENJAMIN. A Compendious and Methodical Account of the Principles of Natural Philosophy. Ed. 2. 1730.
ZELLER, WERNER AND FRANKE, ALEXANDER. Das Physikalische Rüstzeug des Ingenieurs. 1956.

PLASTICS

GIBELLO, HENRI. Le Styrène et ses Polymères. 1956.

THE FRANKLIN INSTITUTE MUSEUM
A. C. CARLTON, DIRECTOR

A NEW ELEMENT

BY

THOMAS COULSON

An addition must be made to the large Periodic Table of the chemical elements in the Museum to bring it up to date. The discovery of the tenth man-made element has been announced as the result of a research effort of an international team of scientists.

For a number of years we were accustomed to regard the material universe as consisting of ninety-two elements, beginning with hydrogen, the lightest, and ending with uranium, the heaviest. In addition, nature had created hundreds of variants, called isotopes, which contain the same number of protons in their respective nuclei, but differ in the number of neutrons.

In the course of the earth's billions of years of existence, four of the original ninety-two elements have disappeared as a result of radio-active decay. These are technetium, promethium, astatine, and francium. The gaps have been filled by new transmutations. The transmutations are a modern version of the alchemist's dream, although it must be claimed the methods show a considerable divergence.

Most of the modern alchemy dates from the year 1940 when the first man-made element, neptunium, was created by bombarding uranium with neutrons fired from a giant cyclotron, transmuting element 92 into element 93. Neptunium transmuted itself into element 94, plutonium, the element used in one of the atomic bombs. Once the way had been found, it became possible to extend the number of man-made elements until we have now arrived at number 102.

A certain romance surrounds the search to identify many of the natural elements, but the discovery of the man-made elements follows an entirely different pattern. They are all the result of skillful research in which each step is carefully worked out toward a predetermined end. After the discovery of neptunium and plutonium, the next four elements were discovered in the Radiation Laboratory of the University of California, by groups of scientists working under the direction of Professor Glenn Seaborg, who was awarded the Nobel prize for the work. Elements 99 and 100 were first found in the debris of the atomic explosion at Bikini in 1952, and created by less violent methods in the laboratory at Arco, Idaho. The scene then reverted to the University of California, where element 101 was created by bombarding element 99 with alpha particles (nuclei of helium atoms) thereby adding two protons to the nucleus of einsteinium.

The discovery of the latest element which it has been suggested should be called nobelium after the Swedish chemist Alfred Nobel,

founder of the Nobel prize, has been the result of international co-operation between American, Swedish, and British scientists. The new element has been produced by bombarding element 96, curium, with nuclei of carbon of mass 13, that is a nucleus containing six protons and seven neutrons. The six protons of the carbon were added to the ninety-six of curium to create a nucleus of 102 protons. The cyclotron of the Nobel Institute was employed.

Knowledge of atomic structure has arrived at a stage in which predetermined results may be reached with a minimum of experimentation. A lapse of only a few days followed before indications were observed that the new element had been created. A month later another series of experiments confirmed the original findings, and a third series furnished conclusive proof of the element's existence. The new element is unstable, having a half-life of ten to twelve minutes.

While the Periodic Table in the Museum shows that the greater portion of the chemical elements have useful applications, only one of the man-made elements, plutonium, has industrial potentialities. It has vast potentialities as a nuclear fuel for the production of power as well as for radio-active isotopes for use in medicine, industry, and agriculture. The other synthetic elements are produced in only small quantities and no practical use for them can be foreseen. Apart from the understanding of the forces operating in the material universe to which these man-made elements contribute, they may yet be of high practical value in providing a better knowledge of the common natural elements.

***Seminar on* Uses of Time-lapse Cinematography in the Study of Cellular Activities and Growth Phenomena.**—DR. T. C. HSU of M. D. Anderson Hospital and Tumor Institute, Houston, Texas, addressed the Seminar of the Biochemical Research Foundation on Monday, June seventeenth.

Dr. Hsu made the following remarks as a commentary on his time-lapse film.

For years we have been taking pictures of cells in tissue culture with phase contrast microscopy and time-lapse cinematography. We are able to observe many cellular activities which we ordinarily cannot visualize because we are so used to seeing fixed cells and many of our concepts are built up from such static pictures. For instance, we think of chromosome division in mitosis as starting from the equatorial plate of late metaphase with the homologous chromosomes moving toward opposite poles during anaphase. However, occasionally in certain cells the two groups of chromosomes at the poles may move together again toward the center of the cell and there they may combine to form a single nucleus. Had we examined such cells in fixed preparations we would not have been able to differentiate the two types of movements. The use of time-lapse cinematography thus allows us to see the dynamics of the cell as well as cellular components.

The first time-lapse film is a very recent one showing the germination and early growth period of the mold *Neurospora crassa*. This mold is very widely used by both geneticists and biochemists in the study of biochemical genetics and metabolism.

An analysis of the film reveals that there is a definite relationship between the size of conidiospores and the time required for germination, namely, the larger spores germinate faster than the smaller ones. The ascospores may germinate from both ends or one end only. The first appearance of the protoplasm from ascospores is invariably in the form of a sphere. The mycelium then grows out in several directions and forms branches later. We could analyze the growth-rate by projecting the frames, measuring the mycelial length or area, and computing the time required for each interval by the number of frames. This can be done with various kinds of media and other environmental factors to determine quantitatively and microscopically the effects of environment on growth.

The protoplasmic streaming can easily be seen. The mycelium is divided by numerous septa which contain holes through which the protoplasm can pass. Vacuoles in the protoplasm can be seen squeezing through the hole in the septum.

The mycelium tips from a number of different spores can fuse to form a continuous system. The protoplasm from two different organisms would then be mixed. In such a common protoplasm, many types of mutations would be difficult to detect. Apparently there is some kind of attraction between the mycelia that causes them to grow toward each other and fuse. The mitochondria are usually short, but in some mutant strains they may be long and filamentous in form.

This part of the film shows a strain of Neurospora which produces colonies. This is due to a gene which causes the mycelia to grow in a curved manner instead of a straight line and that is why the colony doesn't spread out very far but forms a small roundish colony. However, there are also many agents which when added to the medium will cause the wild type Neurospora to grow in the form of colonies. One such chemical is l-sorbose. With this treatment the tips of the mycelia burst allowing the protoplasm to flow out and the tip dies, resulting in no further growth at that point. Another mycelium is then produced at some region back of the dead tip but this new mycelium tip, after a short-lived initial growth, also bursts and dies. This happens to all new growth and therefore this spacial limitation results in a colony.

While colonies result from the action of both the colonial gene and the l-sorbose, the manner in which they are produced is quite dissimilar.

The remaining two time-lapse films depict such phenomena in tissue culture as cell movement, normal and abnormal mitoses, rotating nuclei and movement of the mitochondria. (*Summarized by Irving Galinsky.*)

THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

THE SHOCK TUBE—JUST A PIECE OF PIPE

BY
GEORGE PETER WACHTELL *

A shock tube is one of the most versatile tools in the modern physicist's or chemist's kit for the accurate measurement of fast phenomena. Shock tubes have been used to measure how quickly heat can flow from one mode of motion ("translational") to another ("vibrational") in gas molecules. They have been used to calibrate fast pressure gages, to study boundary layers in supersonic flow, and to simulate the effect of blast waves on structures. They have been used to study the interaction between magnetic fields and electrically conducting gases in rapid motion. They have been used to generate extremely high temperature and to examine fast chemical reactions.

To build a shock tube, all one needs to begin with is a piece of pipe. The shock tube put together in the Laboratories for use on some chemical studies for Army Ordnance is a pipe 33 ft. long divided into two sections (see Fig. 1). The "driver" section, 22 ft. long, contains the

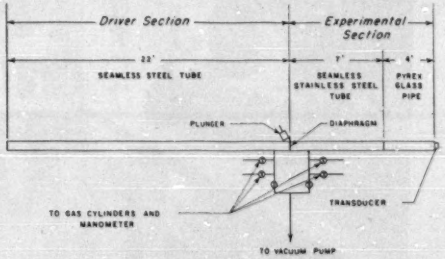


FIG. 1.

"driver" gas (usually helium in our experiments) under pressure, and is separated from the "experimental" section by a cellophane diaphragm. The experimental section contains the experimental gas at reduced pressure. When the cellophane is torn by an electrically operated plunger, there is no longer anything to support the large pressure difference between the two gases. As a result, a shock wave moves at supersonic speed into the experimental gas, heating it, compressing it, and setting it into motion. When the shock wave strikes the closed end of the ex-

* Senior Research Physicist, Chemical Kinetics and Spectroscopy Section. Work carried out on Contract DAI-36-034-501-ORD(P)-40, Department of the Army.

perimental section, it is reflected as another shock wave which further heats and compresses the experimental gas and brings it to rest.

The shock wave may be crudely visualized by imagining a wall of water advancing over the countryside after the sudden bursting of a dam. The depth of the moving water corresponds to the gas pressure after the shock wave has passed. If the water collides with a steep mountain, it will be brought suddenly to rest. The moving water behind it will "pile up" and the result will be water at rest but deeper than it was before, and another wall of water moving away from the mountain. Despite the cataclysmic analogy, however, the shock tube is almost completely silent in operation. The only sound is the click of the plunger that tears the cellophane.

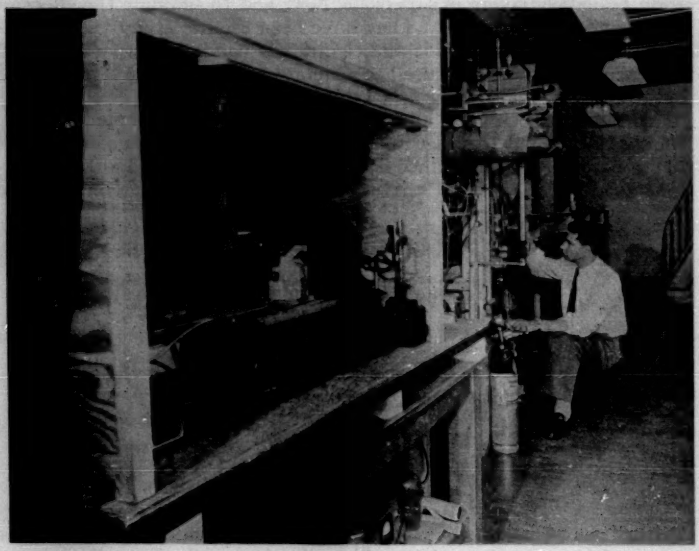


FIG. 2.

The shock tube at The Franklin Institute (see Fig. 2) has been used to measure the temperature and pressure at which certain combustible gas mixtures will ignite, and the "ignition lag" between reflection of the shock wave and the onset of visible flame. The temperature and pressure generated by the reflected shock wave have to be calculated from data which include the speed of the first shock wave. This speed is found by measuring the time between the instant when the shock wave

passes a light beam and the instant when it strikes the end of the tube. Ignition is observed by means of a photomultiplier. Because of this use of optical devices, the last four-foot portion of the experimental section is made of Pyrex glass.

To record the data requires two oscilloscopes, two cameras, and a multitude of special electronic circuits. Preparation of an experiment requires a vacuum pump, a flask for mixing the experimental gases, and a maze of valves and tubing. Although the shock tube is just a piece of pipe to begin with, the complexity of the auxiliary equipment may lead one to suspect that perhaps it is the shock tube that should be regarded as "auxiliary."

BOOK REVIEWS

INTRODUCTION TO MATHEMATICAL LOGIC. VOL. I, by Alonzo Church. 376 pages, 6 × 9 in. Princeton, Princeton University Press, 1956. Price, $7.50.

There are three main objectives which a review of such a book as this may strive to reach. In the first place, a review should inform a reader who has not seen the book as to its general nature and contents, so that he may decide whether to read or to buy it. In the second place, it may try to help those who do read the book to derive greater benefit from the experience. In the third place, it is the duty of a reviewer to suggest improvements and to point out the inadequacies, so that later workers may be deterred—if that is possible—from mimicking the faults of the book while doing justifiable homage to its virtues. The present reviewer is attentive to all three of these aims; and the reader, should he wish to make value judgments on the basis of what is written here, is cautioned to bear this in mind.

It is necessary to begin with a few words about the author; for this is one of those cases in which it is important to understand the author if one is to understand his book. He is one of the world's best-known logicians. For over twenty years he has been one of the editors of the *Journal of Symbolic Logic*—indeed he may well be regarded as the founder of that journal, and, at least in the initial stages, the principal determiner of its policies. This *Journal* has become internationally famous; and the review section, for which he is particularly responsible, especially so. In the first volume of the *Journal* he published a "Bibliography of Symbolic Logic" which is one of the most comprehensive ever published for a mathematical subject. He has made important contributions to mathematical logic; and some of his former students are now leaders in that field. By nature he is phlegmatic and methodical; his work is characterized by meticulous precision, indeed his capacity for attention to minute detail is phenomenal; and his knowledge of the literature of mathematical logic probably exceeds that of any living person.

Under these circumstances it is natural that the book should be a work of scholarship. This is indeed a book for those who want a full treatment, with adequate attention to the nicest of technicalities. Readers who want a quick summary of the main ideas, with a minimum of detail, should look elsewhere. The preface states, to be sure, that the work is intended as a textbook for a beginning course in mathematical logic for students with "some substantial mathematical background." However, such students would probably derive greater benefit if they read some more elementary book on the subject first. The reason for this is not that the book is not clearly written—quite the contrary—but the order of presentation and amount of detail are not well suited for beginners.

As a work of scholarship, however, it is a magnificent achievement. No one who aspires to become a master of mathematical logic can afford to ignore it. Not only does it contain a great deal of information not easily available anywhere else, but it is also valuable for its expository material and critical comment. The major results of the parts of logic treated are presented in masterly fashion. There are extensive lists of exercises: some of these are straightforward illustrations for practice; but a large number of them are results, supplementing the text, taken from the published literature—in one case the student is asked to prove Gentzen's "Hauptsatz," the proof of which occupied 14 pages in the original paper! For the expert, one of the most valuable features is the wealth of historical comment. The numerous critical remarks are, of course, from the author's point of view, which not everyone will agree with; but whether one agrees with them or not, they are worth thinking about. The work, like every human document, has certain faults; but by and large this book will doubtless become one of the classics of the logical literature.

The book is the first volume of a projected two-volume work. It deals with various forms of the propositional calculus and of the predicate calculus of first-order (which the author insists on calling the "functional calculus"), and of the calculus of second-order; also, in the introduction, it discusses various general principles of formalization, symbolic usage etc. It is an amplification of a monograph of the same title which the Princeton University Press published in 1944. The amplification is considerable, because the earlier work had 118 pages reproduced by photo offset, whereas the present work has 376 pages of letterpress printing; but passages from the earlier work have been reproduced verbatim. It was a long time in process; although the copyright date is 1956, the preface is dated 1951. The meticulousness of the author is shown by the fact that only in a small number of footnotes marked "added in proof" is there any evidence that the work has been influenced by events since 1951; so that the book had somewhat the appearance of being out of date when it first came off the press. One cannot help noticing that there are many more references, in proportion, to work done in the 1930's and 1940's than to that done in the 1950's.

The book consists of the main text, the exercises, and 590 footnotes numbered serially throughout the entire volume. The exercises, in the aggregate, comprise some 70 pages, and the footnotes probably about the same. These two sorts of supplementary material add a great deal to the value of the book. One wonders, however, why the author did not devise some system of numbering the footnotes section-wise, as he did for the theorems and exercises; and whether such a device would not have brought the book out sooner.

It is now necessary to point out certain defects of the book, and to indicate ways in which it seems to be biased.

There is no treatment of the algebraic approach to logic, and only incidental mention, in the exercises and historical remarks, of the constructive or intuitionistic systems. Now many of the properties of logical calculuses are common to a variety of essentially different ones, and one could follow the practice of modern algebra in recognizing systems (or, if one will, language-types) of intermediate grade, like rings and integral domains. Thus one could develop the properties which intuitionistic and classical systems have in common first, and then, since one is interested in the latter, one could introduce the more special assumptions of the classical case at a later stage. The author suggests doing this in the exercises. The contention made here, however, is that it should be done in the main text, whereas the derivation of the rules of the resulting system from the basis of the text should be relegated to the exercises; the reason is that there are significant applications of the intuitionistic systems in, for example, the domain of recursive functions. But the reviewer would go even further, and organize the treatment into various levels of generality. Thus the principle of duality and the "substitutivity of equivalence"—an archaic term, which ignores completely the distinction proposed by Carnap between substitution and replacement—are properties of systems of very general type.

A related criticism is that the inferential rules of Gentzen are rather slightingly treated. The reviewer believes that it would be advantageous to make more extensive use of them.

The author's use of the terms "semantical" and "syntactical" seems to the reviewer, and to some of his friends, rather strange. The reviewer has argued elsewhere that these terms are vague, and that they overlap with one another. Consequently, it is understandable that the author should interpret them in a singular way; the reader should make allowance for that usage.

Again the references to the literature are primarily concerned with the origins of ideas—with assigning credit where credit is due. But there is another consideration in making references, *viz.* helping the reader to find facts. Original sources are not always the best places to consult for subject-matter information. Now the references to sources are valuable, especially for the expert. However, when the original source is inaccessible, or differs markedly from modern notation, or is otherwise unsuitable for current consultation, it would help certain readers to give references to secondary sources or expository material. The author does this in a few cases, but the reviewer noticed the lack in some cases with which he is familiar, and wonders whether there may not be others. It may even be that this preoccupation with

sources distorts the history, so that there may be a germ of truth in the remark of a friend that the author is more of an archaeologist than a historian.

Again, the book would be more useful for references if the source of the exercises were stated at the point where the exercise occurs. This is now done with a few exercises; but in other cases one has to search for the reference in the historical notes several pages further on.

The author's continuance of the word "disjunction," in spite of the fact that disjointness is no longer associated with it, is unfortunate. To be sure the word is sanctioned by habit, but a proposal to change that habit has been make and deserves support.

Finally the reviewer wishes to protest against a certain affectation in the author's language. Two cases are singled out for comment. These two are "calculi" for "calculuses" and "singulary" for "unary."

In regard to the first, the Latin word "calculus" means a small stone or pebble. When it is used in English in that sense, as it is in medicine, it is quite proper that its plural be formed according to the rules of Latin grammar. Its mathematical use is derived from this by metonymy; and although there seems to have been, even in Cicero, a transition stage, its modern use has departed so much from the primitive meaning that we are no longer conscious of any connection with "calx." Moreover, in this modern use there was no need of a plural for a long time. Under these circumstances the insistence on "calculi" seems pedantic; and along with "formulae," "lemmata," "schemata" it should be banished as an unnecessary archaism.

In regard to "singulary" the author says that he follows Quine "in adopting this etymologically more correct term, rather than the presently commoner 'unary.'" Now this preference for "singulary" seems ill-founded, for the following reasons. In the Indo-European languages there are two roots for "one." From the first root are derived Greek οἶνός; Latin "unus"; Russian "odin"; German "ein"; English "one." From the other root we have Greek εἷς, μία, ἕν; Latin "semel," "simplex"; Russian "sam"; German "(zu)-sam(men)"; English "same", "some." Classical Latin formed "singularis" from the first of these roots; and post-classical Latin formed "singularius." The latter seems to have been merely a variant of "singularis"; and the common etymological descendent of both of them is not "singulary," but "singular." The latter word, however, developed semantically in a singular way, so that it was not felt as parallel with "binary," "ternary," etc. When the need arose for a new word, it was natural to form "unarius," especially since, as a prefix, "un-," "uni-" was parallel to "bi-" even in classical Latin. An obscure reference, found at Louvain, indicates it was so used by Valerius Probus in 1478. It therefore seems plausible that it has been in use—restricted, to be sure—for five centuries. If so, it has a better etymological justification than "singulary." But even if one does not agree with this, etymology is not the only criterion of usage. Do Church and Quine, when they wish to ride in a certain type of four-wheeled vehicle, insist on calling the contraption an "autokinetic," preferring this etymologically more correct term to the presently commoner "automobile"? The use of "singulary" is again a bit of pedantry which ought not to be carried further.

These criticisms do not affect the judgment that this book is of outstanding value. Let us hope that it will exert a great influence in the future; but it ought not to be followed blindly.

HASKELL B. CURRY
The Pennsylvania State University

MATHEMATICS AND COMPUTERS, by George R. Stibitz and Jules A. Larrivee. 228 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $5.00.

The brief title *Mathematics and Computers* aptly describes the subject matter of this book. To be somewhat less brief, "Applied Mathematics and Automatic Computers" would be more precise. Computers of both the analog and digital varieties are treated, with emphasis on the nature of the types rather than on specific machines.

The reader will breeze through the first chapter hardly realizing he has encountered many terms of the mathematician's trade, such as rational and irrational numbers, transcendental numbers, real and complex numbers, function, root, limit, integral, derivative

and differential equation. Each is mentioned only briefly and in a loose informal manner. The need for applied mathematics is then developed; and the authors, mathematicians of stature, let the outside world in on a bit of family history concerning the differences in the points of view of the pure and applied members—a skeleton usually kept hidden in the family closet, though well-known to all mathematicians and engineers. This all leads up to how computation fits into the over-all picture and what gives rise to the need for it.

Types of problems for computers and their origins precede a discussion on the history of the devices themselves. The methods applied are treated in a chapter on numerical analysis which is a subject by means of which the mathematical problem is broken down into fundamental operations. The digital computer components which perform these operations are explained and unity is achieved by tying these together with an ensuing chapter on logical design. At that point material is inserted on analog computers and simulators, after which Drs. Stibitz and Larrivee discuss "Computing with Random Numbers," "Computer Errors" and wind up with "Computers at Work."

Most of the chapters are followed by several "Suggested Reading" references, and a selected bibliography is included as an appendix. A further bibliography classified according to field of computer application concludes the appendices.

The authors have intimate knowledge of the subject matter treated. Dr. Stibitz is a former member of the staff of the Bell Telephone Laboratories and conceived the basic idea of using telephone switching equipment for computing purposes long before the advent of the present large-scale devices. He serves at present as a consultant in applied mathematics. Professor Larrivee has been active in the field of applied mathematics for over 20 years. Their book is not a detailed text for the computer expert, but rather a book for the layman or mathematician or engineer with no knowledge of computers who wants a more realistic picture of this relatively new and fast-growing field than he can find in the popular press.

DONALD B. HOUGHTON
The Franklin Institute Laboratories

TRANSISTOR CIRCUIT ENGINEERING, edited by Richard F. Shea. 468 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $12.00.

During the brief history of the transistor several notable textbooks on transistor circuits have made their appearance. All of these have been produced with the aid of organized author teams. The first of these was published in 1953 under the editorship of Richard Shea who was also a co-author. The other authors were also members of the staff of the General Electric Company. The next comprehensive book on transistor circuits was produced by a group at RCA in 1955. In that same year Shea wrote a book of his own on transistor audio amplifiers. Now another group of G.E. authors under the leadership of Richard Shea has assembled the latest circuit theory and practice in this extremely important new text.

The eight co-authors who have collaborated with the editor to produce this volume are all specialists in some branch of transistor applications, high frequency circuits, oscillators, power transistors, et cetera. Through their published papers all are known to engineers in the field. We therefore have a text representing the latest circuit developments of recognized authorities practicing in this type of work.

Exploiting the current popularity of the transistor, a number of books have appeared containing collections of previously published papers and data simply paraphrased and redrawn. Such books are less of a contribution since they do not have the authoritative strength imparted by authors with whom the material is mostly original. The methods of circuit analysis presented in the chapters of the new book are largely those developed and used in the daily work of the authors. This is further attested by the clarity and simplicity of the text.

The coverage of this book is not restricted to communications, computers, control circuits or any particular engineering specialty. On the contrary, anyone in a position to utilize transistors in any circuit application will find useful analytical methods described. These methods tend to be symbolic rather than graphical. This approach, however, seems to have so far met with the preference of most engineers in the field.

One who was originally trained in the use of vacuum tubes and the design methods applied to tube circuits will not fail to be impressed with the greater complexity of transistor behavior. Richard Shea and his co-authors recognize this fact and attempt to show that certain practical simplifications can be made without compromising the analytical methods. The success which has already been achieved in practical apparatus using transistors also shows that circuit design with semiconductor devices is not hopeless.

Omission of any discussion of transistor physics has saved much needed space in this volume for its important task of circuit engineering. There are already numerous adequate texts on the theory of transistor action in semiconductor systems, and much is gained by this omission.

Naturally this book is highly recommended for all working with transistor circuits. To those engineers who must re-educate themselves in the transition from tubes to transistors, the book can also serve as an excellent self-teacher. Good chapter bibliographies and problems as well as a few worked-out numerical examples in the text make it equally appropriate for college EE courses.

C. W. HARGENS
The Franklin Institute Laboratories

A HISTORY OF LUMINESCENCE, by E. Newton Harvey. 692 pages, 6 × 9 in. Philadelphia, The American Philosophical Society, 1957. Price, $6.00.

The importance of luminescence at the present time stems from the fact that it is fast becoming the universal method of illumination, and because of its adoption for the television screen. In the recent past it has played a significant role in the discovery of X-rays, electrons, and radioactivity. Hitherto, there has been no adequate treatment of the historical aspects of the subject, showing how our knowledge has developed. Professor Harvey's work fills a conspicuous gap in the history of heat and light, and fills it admirably.

As the jacket on the book reminds us, incandescence has been the universal method of practical illumination for centuries. Incandescence is produced by heat, contrary to luminescence, which is "cold" light, such as we see in the glow of phosphorus and certain solids when exposed to sunlight or various forms of radiation, the bioluminescence of many living organisms, the fungus light of decaying vegetable matter, and the bacterial light of dead flesh and fish. Although luminescence did not find any practical applications until very recently, it was by no means a neglected study, as this book shows.

The volume, a substantial one, deals with the history of the subject down to the end of the nineteenth century. It is divided into three parts. Part I deals with the increasing knowledge of the subject, part 2 treats with the luminescence of non-living matter, and part 3 deals with the luminescence of living organisms. The author presents the views of scientists through the ages in their own words, like a source book. This method has much to commend it for it is only by such means that the reader is given a conception of the contemporary beliefs, not only upon luminescence, but also upon heat and light in general.

The reader will be surprised to learn how prominently the problem of luminescence figured in the speculation and investigations of physicists as they strove to furnish satisfactory theories on heat and light. The author, a former professor of biology at Princeton University, has devoted almost forty-five years to the study of his subject, and he does not appear to have overlooked any of the original sources of information. A 68-page bibliography furnishes conclusive proof, if any were needed, of his persistent search for material, and he has exercised a wise judgment in the selection of quotations. The original comments introduced to provide a well-knit history are scholarly and illuminating.

Nothing but unstinted praise can be offered for a work so thorough and so comprehensive. It should be read by every student of heat and light who has the least curiosity to learn how our present concept of luminescence has been formed.

T. COULSON
The Franklin Institute Museum

LIGHT SCATTERING BY SMALL PARTICLES, by H. C. van de Hulst. 470 pages, diagrams, 6 × 9 in. New York, John Wiley and Sons, Inc.; London, Chapman and Hall, Ltd.; 1957. Price, $12.00.

In much the same manner as particle scattering is used in nuclear physics to measure

properties of nuclear matter, the scattering of light may be used to answer questions regarding the size, shape, and internal light propagation properties of larger pieces of matter. The interest in light scattering stems from its many applications in physical chemistry, meteorology, and astronomy. As in all applications, a well developed theory is essential.

Fortunately in the theory of light scattering the chief problem of interest, the scattering of electromagnetic radiation from a homogeneous sphere, has been solved exactly. However, inasmuch as the parameters vary by a factor of about 10,000 over the range of interest, the computational problem is enormous. It is essential to have various approximate means of handling the scattering problem. These approximation schemes range between geometrical optics on the one hand, where the wave propagation effects appear as small deviations from rectilinear propagation, to Rayleigh scattering on the other hand where no geometrical effects remain.

Dr. van de Hulst's book provides a critical review of the entire field of light scattering theory and technique insofar as the processes of light scattering in large scale agglomerations of matter are simply connected with those processes in the individual particle. The omission of multiple scattering processes and coherent scattering by many particles permits the author to present an exhaustive review of the single scattering process.

The first part of the book, "Basic Scattering Theory," provides the formalism or framework necessary to the description of the scattering of light from a medium consisting of a collection of arbitrary shaped pieces of dielectric. From the viewpoint of this book the various terms, diffraction, reflection, refraction, and surface waves, are considered to be aspects of the general scattering problem.

The second section, "Special Types of Particles," comprises the largest part of the book. It is devoted to a discussion of the rigorous electromagnetic scattering theory for spheres (Mie solution) together with the various approximations that are useful in practice such as Rayleigh scattering; the small phase shift approximation variously called the Born approximation or the Rayleigh-Gans approximation; the multipole expansion into electric dipole, magnetic dipole, electric quadrupole, etc. (optical resonance region); the perfect reflection approximation, and the geometrical optics approximation.

In general the mathematical arguments are schematic and detailed derivations do not impede the flow of ideas. A complete list of references is to be found in each chapter so that full details may be found if needed. Generally, all the important results are given so that, unless one is interested in a specific mathematical detail, the discussions are quite complete.

This reviewer feels that the rigorous solution (Mie solution) has not been formulated with all the clarity that is possible had greater use been made of the concept of logarithmic derivative. Although the first use of this concept was in the field of electromagnetic wave scattering, the clearest formulation of the logarithmic derivative appears in the corresponding scattering problems of acoustics and neutron physics. In particular the author has overlooked completely the connection between the scattering phase shift and the corresponding logarithmic derivative.

The third part of the book, "Applications," deals with the areas to which the theory of light scattering has direct application. The discussion pertaining to each specific technique is cross-referenced to the theory section, thus permitting the experimenter to find most easily the section of the theory of direct interest. The areas of application include physical chemistry, although this topic is not covered fully since the author has limited his topics to those in which the single scattering effect is predominant. Other areas in which the light scattering technique is reviewed include meteorology and astronomy.

Taken as a whole the book by van de Hulst is a scholarly effort with well over 400 references, 46 tables, 59 graphs, and 44 illustrations. The manifold aspects of the theory are treated with pedagogical skill, thus permitting a student to become absorbed in this most interesting field. Certainly the book is a must for researchers using the techniques of light scattering since it provides a critical review of all of the important work in this field.

S. C. SNOWDON
Bartol Research Foundation

DISCOVERY OF THE UNIVERSE, by G. de Vaucouleurs. 328 pages, plates, 5½ × 8½ in. New York, The Macmillan Co., 1957. Price, $6.00.

Gerard de Vaucouleurs is a young, brilliant French astronomer whose fame and reputation are international. He is the author of an authoritative book "Physics of the Planet Mars," the ranking work on this planet.

However, in "Discovery of the Universe" he does not match his previous performance. Perhaps the reason for this is the tremendous scope of his subject matter. In a book of slightly more than 300 pages the author attempts to portray a history of astronomy from its origin, going back perhaps 6000 years, to the present day. In doing this he has expertly trimmed the subject matter and has skeletonized broad facets of the field. Despite this heroic effort he has not succeeded in doing what he has set out to do. This reviewer believes it impossible in the limited number of pages to review the history of astronomy.

There are some favorable aspects of the subject as presented in this book. The author has traced the major developments in astronomy and has so completely, but briefly, covered this that the book has considerable merit as a reference book. The author has wisely included both an author index and a subject index to increase the book's value for this purpose.

In the first eight sections Dr. de Vaucouleurs brings us up to the present day of astronomy. These are rather conventional sections. Where the author really displays his potentialities is in the last section, "Astronomy Today." His account of instrumental advances, solar system physics, galactic and extra-galactic studies, stellar physics and radio astronomy is superb. Had the author just expanded this section and treated only present day astronomy the resulting book would have filled an important need in the field. The reviewer believes Dr. de Vaucouleurs has wasted his talents in writing of historical astronomy which has had comprehensive treatments in the past.

In the foreword Dr. de Vaucouleurs states: "I have often felt, too, that many popular accounts are unbalanced and uncritical in their selection of material. A recital of the obsolete speculations of the Ancients . . . may have some appeal for those interested in details, but they deserve only little space in a volume which aims at giving its readers a comprehensive survey of the whole history of astronomy in a few hundred pages." This book was written so that the integrity of this theme was not violated.

Discovery of the Universe is a well written book but this reviewer expected more because of the author.

I. M. LEVITT
The Fels Planetarium

GAS CHROMATOGRAPHY, by A. I. M. Keulemans, edited by C. G. Verver. 217 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $7.50.

This is the second text concerned with this recently invented but rapidly growing method. The author is an employee of the Shell Oil Company, whose chemists have contributed much to the development of gas chromatography. As such, he has had access to vast amounts of previously unpublished results from Shell's laboratories in Holland, England and the United States, as well as the authoritative views of his colleagues.

It is a much more detailed book than the first one by Phillips. Frequent reference is made to the literature and many of the publications which appeared as recently as June, 1956 are discussed.

Chromatographic techniques are briefly described and classified in the first chapter. The factors affecting separations are discussed thoroughly, but qualitatively, in the second chapter. Analytical applications with important examples are cited. In Chapter 3 the apparatus is described. Not only the common, but several interesting, non-commercial detection devices are mentioned. The author presents the theories of gas chromatography in Chapters 4, 5, and 6 and, in Chapter 7, shows how these principles can be utilized when dealing with difficult separations and non-analytical applications. Chapters 2 to 7 are concerned primarily with the more important method of gas-liquid chromatography, while Chapter 8, the last one, deals with the older method of gas-solid chromatography, which possesses advantages in a few specific cases.

Many separations and analyses may be made by gas chromatography without any

knowledge of the quantitative aspects of the theory. The author has kept this in mind in the arrangement of the subject matter. An adequate working knowledge of the method can be gained by a study of the first three chapters. The experimental techniques and operating variables are so thoroughly covered in these sections of the book, that familiarity with this material is generally sufficient to overcome the difficulties encountered in routine applications. Those investigators concerned with more difficult problems will find the theoretical discussions quite helpful. These principles have usually been illustrated by interesting experimental data.

This book is the most complete single reference on gas chromatography. As the inventor of the method, A. J. P. Martin, wrote in the foreword, "Dr. Keulemans demonstrates in this volume his mastery of the subject which reflects the vast amount of work which he and his colleagues have put into the development of this analytical tool."

MANFRED PRAGER
The Franklin Institute Laboratories

AN INTRODUCTION TO SEMICONDUCTORS, by W. Crawford Dunlap, Jr. 417 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $11.75.

The periodical literature on semiconductors has grown to an enormous size. While very few books have been written on this subject (one could mention the excellent works of Shockley and of Henisch), each of these puts a special emphasis on a particular aspect of the field. Shockley, for example, gives a thorough elucidation of the theoretical foundations and deemphasizes the applications and devices. Thus, there has been for some time a great need for a general synthesis of the subject at an intermediate level. This need has been successfully filled by Dr. W. C. Dunlap's book. Without undue emphasis on the theory, Dr. Dunlap's treatment is vigorous and will satisfy both the physicist and the engineer who wish an introduction to the subject. Those already working in this field will find all the useful ideas and facts gathered and exposed in a clear and logical fashion.

In the first eight chapters, Dr. Dunlap gives an exposé of the essential Solid State background necessary for the study and understanding of semiconductors. The following two chapters contain the experimental facts or measurements and preparations of semiconductor materials. The properties of semi-conducting elements and compounds are thoroughly analyzed and Chapters 11 and 12, with a legitimate emphasis on germanium and diamond. The rest of the book (Chapters 13 to 15) is devoted to the study of the various semi-conducting devices, transistors, rectifiers, thermistors, photocells, etc.

Among the features to be particularly commended, let us mention: (1) the three-dimensional treatment of the energy surfaces, more realistic and more rigorous than the standard E-k Diagram, often misleading (especially when the direction of vector k is not specified); (2) the excellent chapter on Statistical Mechanics; (3) the list of pertinent references given at the end of each chapter. In conclusion, we feel that Dr. Dunlap's book will prove to be a good text book for the student and a useful reference for the worker in the field of semiconductors.

HENRI AMAR
The Franklin Institute Laboratories

NEUTRON TRANSPORT THEORY, by B. Davison, with the collaboration of J. B. Sykes. 450 pages, diagrams, 6 × 9¼ in. New York, Oxford University Press, 1957. Price: $12.00; 75 s (U. K. only).

Do technical reports become published papers? A report on this question appeared in the June, 1957, issue of *Physics Today* in which D. E. Gray and S. Rosenborg studied what happened to the publishable material in 1018 unclassified technical reports. One of their conclusions was "that for about a fifth of the reports that contain publishable data, none of the information is published, at least for several years, and much of it probably never appears in conventional printed form." When we consider also that many of these technical reports, although unclassified, are completely unavailable to most scientists, we realize that much valuable information may be lost to the scientific community.

As one who has been an active participant in the advancement of neutron transport theory, B. Davison has recognized that many of the significant contributions have been available only in technical reports, and some have not appeared in any form. In order to remedy this situation, he has gathered to-

gether in this book all the principal mathematical methods used in neutron transport theory. In order to do this, he has included material from twenty-six American, British, and Canadian technical reports, as well as previously unpublished contributions from Bethe, Elliot, Feynman, Clark, and Serber. Aside from rescuing some of this work from oblivion, the author, with the collaboration of J. B. Sykes, has given a unified presentation of the various theoretical methods and has discussed their regions of applicability.

The author's approach to the various subjects throughout the main body of the text is more mathematical than physical. He prefers to begin with a precise formulation of each problem—rather than starting with a simplified physical picture, deriving the more elementary approximations, and then proceeding to the refinements. Besides the material contained in Whittaker and Watson's *Modern Analysis*, the reader of this book is presumed to know the definition, inversion formulae, and elementary properties of Fourier, Laplace, and Mellin transforms. Thus we see that this book is primarily written for theoreticians, being generally at the level of difficulty of theoretical papers in *The Physical Review*. There are very few diagrams or tables throughout the book, and few, if any, numerical examples for the theoretical methods, which may have been omitted because of space limitations. No comparison is made with experimental studies for reactors or critical assemblies, although much of this material may not have been declassified at the time of writing this book. The result is that the book is rather difficult reading for one not possessing a considerable theoretical knowledge of the subject.

The book is divided into four parts. In Part I, the basic transport equation is derived from the relevant physical properties of neutrons. Part II deals with the constant cross section approximation. Here the various methods for obtaining exact solutions for a few particularly simple problems are described. For more complex problems, the various approximate methods are considered, including the spherical harmonics method, perturbation and variational methods, and the iteration and Monte Carlo methods. In addition, the Serber-Wilson method is discussed, together with the accuracy and limits of applicability of the method. Energy-dependent problems with spectrum regeneration are considered in Part III, with a critical discussion of the assumptions underlying multigroup theory. In Part IV, showing-down problems are dealt with, including a treatment of age theory. The problems of slowed-down neutrons at large distances from the source, for both the cases of constant and variable cross-sections, are analyzed.

This book should be a valuable reference for all those interested in the theory and design of reactors, particularly those involved in theoretical calculations. The description of the approximations involved in each theoretical method, and the evaluation of the usefulness of each method are particularly worthy of attention. The author has rendered a valuable service in making available such a large amount of theoretical material, and by presenting a unified picture of the many contributions to the problems of neutron transport theory.

D. M. VAN PATTER
Bartol Research Foundation

MICROPHOTOGRAPHY, by G. W. W. Stevens. 326 pages, plates, 5¼ × 8¼ in. New York, John Wiley & Sons, Inc., 1957. Price, $8.50.

This is a book on photography at extreme resolution. The importance of this subject is very great today because we are constantly searching for improved methods of information storage. In fact, the whole process of storing information of all types so that it may be quickly retrieved and put to use either in commerce, science, libraries or for whatever purpose is receiving a great deal of attention.

So far the most compact form of stored information is photographic, and techniques on the order of ultra-microfilm have been developed. Perhaps the most notable complete system for automated storage and retrieval employing the principle of microphotography is Eastman Kodak's "Minicard" project. This is briefly mentioned in the book.

Dr. Stevens is a Fellow of the Royal Photographic Society and has performed his researches at the Kodak Research Laboratories, Wealdstone, England. In this work he has had some association with Dr. C. E. K. Mees.

Specifically the technical content of the book relates to sensitive materials for extreme

resolution, processing of microphotographs, optical requirements, the production of reticles, document microphotography, and the potential use of these techniques in scientific research generally. The subject of reticles includes precise code wheels for analog-to-digital conversion. Such devices, made by microphotographic processes, are important in the computing machine field.

With typically British scholarship, Dr. Stevens has dealt with these subjects in a most interesting way, bringing out some of the historical and experimental background of each. In fact the author goes so far as to suggest some basic experiments which the reader can perform to demonstrate certain pertinent facts to himself. These can be done simply and with common materials.

This is an excellent reference text illustrated with many drawings and photographic examples. Its reference value rests, moreover, upon some very good chapter bibliographies which refer to the important proposals in this field by V. Bush, L. N. Ridenour, and others. Not only is the book a thoroughly sound technical work, but it also reflects the highly satisfactory career of its author in an interesting field of activity.

C. W. HARGENS
The Franklin Institute Laboratories

ROCKETS, MISSILES AND SPACE TRAVEL, by Willy Ley. Revised edition, 528 pages, illustrations, 6 × 8¼ in. New York, Viking Press, 1957. Price, $6.75.

The most definitive book in the space travel field has just been revised and released. Since 1944 Willy Ley has carefully kept his book current so that in this newest edition everything worth knowing about this field is included.

The first 170 pages are untouched for they deal with history and the present cannot influence the past. After this section the author, in his superb narrative style, relates the story of Peenemunde. This has been thoroughly covered by General Dornberger in his book "V-2" but this section also contains the background of other important figures at that station. The story of Peenemunde is shaping up as one of the most fascinating stories of World War II. Perhaps at some future date Wernher von Braun will also write about this place from the engineer's point of view.

Following this comes the section on White Sands with the details of the firings of the V-2 and the new Vikings. Then the rocket is treated as the instrument to penetrate cosmic space.

The author covers all the modern facets of this science. He speaks about Project Orbiter, Project Vanguard and even the remarkable performance of the Jupiter C which broke all distance and altitude records. Cognizance is taken of projects of the future and figuring prominently in these plans are the papers given at The Franklin Institute Symposium on "Earth Satellites as Research Vehicles" in 1956.

Rockets, Missiles and Space Travel is the answer to any young man's dream about a book on this vital subject.

I. M. LEVITT
The Fels Planetarium

ELEMENTS OF GASDYNAMICS, by H. W. Liepmann and A. Roshko. 439 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1957. Price, $11.00.

This book is the eighth in the GALCIT Aeronautical Series, edited by Theodore von Karman and Clark B. Millikan, and deals with the fundamentals of gas physics, thermodynamics and chemical kinetics as applied to the problems of high speed compressible and viscous flow. With this volume the authors have provided the engineers and scientists working in problems of high speed aerodynamics with an up-to-date companion to an earlier volume in the GALCIT Series, "Introduction to Aerodynamics of a Compressible Fluid," by Liepmann and Puckett.

The authors place special emphasis on concepts from thermodynamics and chemical kinetics in treating the gas dynamics of real gases in this volume. The aerodynamicist is thus provided with a concise reference for fashionable work in the realm of the flow of high temperature, real gases. This reader especially enjoyed the use of a common model, that of Couette flow, in illustrating separately the roles of viscosity and thermal conductivity, the effects of gas dissociation, and rarefied gas effects on skin friction and heat transfer.

Chapter 6, devoted to methods of measurement, is a particularly valuable one not usu-

ally found in references in this field. The most modern techniques of making aerodynamic and heat transfer measurements in a supersonic airstream are carefully described along with a description of the underlying physical principles. Shock tube instrumentation is too briefly dealt with, however.

It is pertinent to mention that slender body theory, the theory of flow in ducts and wind tunnels, the theory of shock tubes, transonic flow theory, and the similarity rules for high speed flow are all described and illustrated in this book. A fitting closing chapter is devoted to the kinetic theory of gases, probability theory, Boltzmann distribution functions and their place in gas dynamics.

Exercises are provided for each chapter. They are suitable for use in senior level courses for aeronautical students or for graduate level courses. References are footnoted throughout and a list of selected references is provided.

This reviewer would not hesitate to recommend this volume for any scientist or engineer seeking a ready reference in the theory and techniques of modern gas dynamics.

WILLIAM H. DORRANCE
Convair

AIRCRAFT GAS TURBINES, by C. W. Smith. 448 pages, diagrams, $5\frac{1}{4} \times 9$ in. New York, John Wiley & Sons, Inc., 1956. Price, $8.75.

This book is one of the General Electric series, covering a course of lectures by the author, both at New York University and the Aircraft Gas Turbine Division of the General Electric Co.

The author covers a comprehensive technical discussion of the theory of the aircraft gas turbine and includes some discussion on design proportions and limitations. However, it would have been hoped that a more detailed blocking out of design proportions would offset much repetition of basic principles already found in the literature on the gas turbine power plant.

In Chapter 2, the author discusses aircraft propulsion theory and introduces such concepts as thrust considered as a pressure integral. While of course, in greater part, the thrust reaction between the fluid passing through the power plant and nozzles is the axial component of the pressure integral over the constraining surfaces, additional forces due to friction must be considered, even though the interacting surface friction forces are usually of small order. However, the use of a pressure integral is of value when considering the net thrust and the significance of the external forces. The author has shown the ambiguity of measurements in the thrust rating of an aircraft gas turbine, and here the pressure integral over the external surfaces offers a clearer picture for defining the net thrust.

The aircraft gas turbine power plant differs from the stationary power plant in the manner of utilizing its energy. For this reason, it would have seemed preferable to discuss the over-all energy equation directly relating the propulsion power with that developed in the power plant, rather than through efficiency ratios and comparative cycles. Moreover, in the author's discussion of the "Brayton cycle as modified by losses," it would seem that the comparison of the actual nonisentropic adiabatic compression and corresponding nonisentropic expansion between definite pressure limits should rather be made with the Brayton cycle as a yardstick. The author's own comparison, in his definition of efficiency of compression, seems adequate, without reference to cycles.

In Chapter 6, the author covers the thermodynamic relations for compression and expansion of the gases as viewed from their properties. It also includes an interesting discussion of elementary gas dynamics for supersonic flow.

In the theory of flow, the author discusses the enthalpy changes along ducts, between entrance and exit sections, extending the nozzle analysis to supersonic and subsonic zones, with a discussion of shock losses.

The author discusses the energy relations for ramming intake. A simplified problem considered by the author is the acceleration of a mass m of the air to the velocity V_p of the airplane by the interaction force F between the airplane and air. With V relative to the airplane in the direction of flow, the absolute velocity of the mass m is $V_p - V = v$ forward. Then $F = m\frac{dv}{dt}$ and the energy input by the airplane is $\int^{V_p} FV_p dt = mV_p^2$. This is the

energy transfer between airplane and fluid of mass m. The energy input to the air by virtue of the force F acting on it is $\int^{V_p} F v dt = \frac{mV_p^2}{2}$ which is the kinetic energy acquired by mass m. The difference is absorbed and is assumed to be that for compressing the air. That is, the work of compression corresponds to the slippage or relative work $-\int_{V_p}^{0} FVdt = -\frac{mV_p^2}{2}$. Thus the energy input from the airplane equals the dissipation or stored energy $\int_{V_p}^{0} FVdt = \frac{mV_p^2}{2}$ plus the kinetic energy $\frac{mV_p^2}{2}$ acquired. This is believed to be the author's interpretation and is useful in giving one aspect for a preliminary idea of the problem.

The actual problem, however, relates to a continuous flow rate and is concerned with the pressures at both the entrance and exit sections of the ram which latter leads to the compressor. While the energy input to the fluid flow passing through the duct is $\int FV_p dt$, the energy input is no longer mV_p^2 per mass m of air, even with relative entrance velocity V_p. The difficulty arises in that F used in the integration is no longer the reaction between the intake flow and airplane, since in addition the terminal pressures reacting on the flow must be considered. Further, the work of the pressure interactions across sections of the flow must be considered, in addition to the work of the airplane reaction, moving with velocity V_p. In estimating the pressure head at exit, an energy balance must also include the change in enthalpy. The power input is then equal not only to the airplane input but also must include the rate of work of the end pressures acting on the flow. The power equation, along with the momentum equation, for the case discussed reduces to its more simple form, in terms of its *relative* energy, where the enthalpy change per unit weight of flow is $i_1 - i_2 = \frac{V_2^2 - V_1^2}{2g}$, and V_1 and V_2 are the *relative* entrance and exit velocities. In general, V_1 is not equal to V_p as brought out by the author in his previous discussion. Possibly the total change in enthalpy per unit weight of flow can be divided into two components, that for the change of the flow to the entrance to the duct, that is, $i_1 - i_0 = (V_p^2 - V_1^2)/2g$, where $i_1 - i_0 = c_p(T_1 - T_0)$ and within the duct, $i_2 - i_1 = (V_1^2 - V_2^2)/2g$, where $c_2 - i_1 = mi_p(T_2 - T_1)$. Since the temperature rise T_2 is thus fixed irrespective of the irreversibility, the corresponding increase of entropy can only lower the pressure p_2 at entrance to compressor.

On the whole this book contains much valuable information and discusses a wide range of general problems pertaining to aircraft gas turbines, and as such it should be useful in a curriculum course and as a well informed practical treatise on the subject.

RUPEN EKSERGIAN
The Franklin Institute Laboratories

BOOK NOTES

THE PAPERS OF WILLIAM FROUDE. 344 pages, diagrams, illustrations, 8½ × 11 in. London, The Institution of Naval Architects, 1955. Price, not given.

Collected and published in one volume by The Institution of Naval Architects, the papers of William Froude, pioneer in ship model research, are presented as a memorial made possible by contributions from organizations in twelve countries. The volume, handsomely printed and bound, comprises all the known published technical papers of Froude, arranged in chronological order from his first, in 1847, on the discharge of elastic fluids under pressure, to the last, in 1878, on the relation between pitch, slip and propulsive efficiency. The last entry consists of two extracts from letters, one written by William Froude shortly before his death in 1878, on the skimming flight of birds, and the other by R. E. Froude, continuing the discussion of the first letter.

The volume opens with a tribute to William Froude by Sir Westcott Abell and an evaluation of Froude's work by R. W. L. Gawn.

CHEMISTRY OF PLANTS, by Erston V. Miller. 174 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $4.75.

The author discusses the chemistry of plants in such a way as to interest research workers in medicine, agriculture, food processing and the manufacturing of chemicals for fungicides, insecticides and phytocides. Twelve chapters cover, for example, carbohydrates (in which gums amd mucilages are treated), proteins, lipides (including essential oils), plant pigments, enzymes, hormones, glycosides, vitamins, etc. References to other works in the field are given at the end of each chapter.

AMERICAN CIVIL ENGINEERING PRACTICE, VOL. III, edited by Robert W. Abbett. 1279 pages, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1957. Price, $25.00.

The third and final volume of this extraordinarily complete handbook covers masonry, reinforced concrete, steel, and timber structures. It contains thirteen sections on broad topics such as theory of structures, prestressed concrete structures, retaining walls, steel bridges, buildings, earthquake-resistant designs, etc. Fifty-four contributors, each an authority in his field, are listed, showing what topics each is responsible for. Each section is amply illustrated and the book contains many tables of valuable data. As in the previous two volumes, the ingenious "edge index" system aids the user to locate sections readily. Paging is by sections.

MODERN APPLIED PHOTOGRAPHY, by G. A. Jones. 162 pages, plates, 4½ × 7½ in. New York, Philosophical Library, 1957. Price, $4.75.

The author confines his treatment of modern applied photography to a very brief outline, with emphasis on the lesser known applications in science and industry; he does not deal with the "art" of photography nor with photography as a science. He has written the book for those readers with a general knowledge of the elementary principles of photography.

Some of the applications dealt with include document copying, scientific recording of images for future study, star photography, infrared photography (medical, high-temperature, aerial, etc.), ultraviolet photography, radiography, the recording and analysis of motion, and the use of photography in production, particularly in inspection.

UNDERSTANDING HI-FI CIRCUITS, by Norman H. Crowhurst. 224 pages, diagrams, 5½ × 8½ in. New York, Gernsback Library, Inc., 1957. Price: $2.90 (paper); $5.00 (cloth).

Aimed at the rapidly growing numbers of hi-fi fans who tinker with their own sets, this book by an authority in the field contains eleven chapters dealing technically with the several components of a hi-fi set. After studying the sections of the book (special output stages, feedback and damping, inverter and driver stages, input stages, matching, equilization, speaker distribution and crossovers, loudness and volume control, and tone controls), the reader should be in a good position to select the components best suited to his purpose. Emphasis is on audio circuits, but the author includes consideration of other factors, such as speakers, cabinets, vibrations, acoustics, etc.

FIVE MATHEMATICAL STRUCTURAL MODELS, by Otto F. Fischer. 412 pages, 5½ × 8 in. Stockholm, Axion Institute, 1957. Price, $10.00.

The interested reader is referred to the review of the author's *Universal Mechanics and Hamiltonian Quaternions* which appears in Vol. 252, p. 446 of this JOURNAL. That review will give the reader a "feel" for the subject matter of this book. The reviewer suggested at that time that a judicial selection in a more highly organized form would enhance the appeal of the material. That apparently is what has been done, to some extent, in this book.

The present book is concerned with the search for structural features in nature which may be expressed in the form of mathematical models. Quaternions are the mathematical instruments used in the search, and five sample models are chosen for study. The features in nature covered by these models range through the complete gamut from mechanics and electromagnetism through to quantum mechanics.

PUBLICATIONS RECEIVED

The Editors wish to call attention to the following books, received in the JOURNAL office for review. Lack of space prevents publication of more than this listing.

WILLIAM HARVEY. HIS LIFE AND TIMES; HIS DISCOVERIES; HIS METHODS, by Louis Chauvois. 271 pages, diagrams, 6 × 9 in. New York, Philosophical Library, 1957. Price, $7.50.

QUALITY CONTROL AND STATISTICAL METHODS, by Edward M. Schrock. Second edition, 246 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $6.75.

COLLEGE CHEMISTRY, by William H. Nebergall and Frederic C. Schmidt. 788 pages, diagrams, 7 × 9 in. Boston, D. C. Heath and Co., 1957. Price, $6.75.

TECHNICAL DESCRIPTIVE GEOMETRY, by B. Leighton Wellman. Second edition, 628 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $5.75.

JOURNAL OF A SCIENTICIAN, by Piero Modigliani. 136 pages, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $3.75.

ORGANIC CHEMISTRY, by Louise Kelley. Second edition of "Organic Chemistry" by G. Albert Hill and Louise Kelley, 757 pages, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $7.50.

Instruments for Satellites.—A tiny new moon, man-made and earth launched, with expected greatest visibility about an hour after sunset and an hour before sunrise will soon be wheeling around the Earth once every 90 minutes according to calculations of scientists engaged in the U. S. Navy's Project Vanguard.

Varian Associates, Palo Alto electronics firm, will participate in this dramatic chapter in scientific progress since their magnetometer, a device which measures the earth's magnetic field, will be one of the instruments carried in a 21½-lb. satellite launched in the Vanguard program. The "artificial moon" is expected to describe an elliptical orbit around the earth from 200 to conceivably 1500 miles out in space. It will circle the earth 15 times each 24 hours, its path crossing the United States in 10 minutes.

A Varian magnetometer in the satellite will provide a time record of the earth's magnetic field above the ionosphere and is expected to answer many important questions concerning magnetic disturbances which have a common origin with disturbances in vital radio communications. Commercially, the Varian magnetometers are used principally by geophysicists to help locate oil and mineral deposits.

Already being used in high altitude rockets for scientific research, Varian's proton free precession airborne magnetometer was selected by the Naval Research Laboratories for their Project Vanguard for its three primary advantages over other magnetometers in applications such as installation in a rocket or satellite. Insensitive to orientation, the Varian magnetometer can be used in any position; needs no calibration; and is the only magnetometer able to transmit its measurements in the form of precession frequency which can be measured and recorded on the ground.

The Varian magnetometer was invented by Dr. Russell H. Varian and constitutes a unique method of utilizing the gyromagnetic properties of the nucleus of an atom. The device itself is ingeniously simple consisting essentially of a coil of wire immersed in a bottle of liquid containing hydrogen atoms such as water, kerosene, gasoline or alcohol. A light hydrocarbon with low freezing point will be used for the satellite flight.

Since protons act as if they were tiny spinning bar magnets, their function can be compared to that of the armature of an electric generator. As they precess like gyroscopes about the earth's magnetic field, these protons induce a current in the coil of wire surrounding them just as the rotating magnet or armature of the generator induces a current in its coil. Actual measurement of the magnetic field is made by determining the frequency of the current induced in the coil by the spinning hydrogen nuclei. This frequency is directly proportional to the strength of the earth's magnetic field. The current induced in the Varian magnetometer in the satellite is expected to range in frequency between 800 to 2200 cycles while circling at varying heights above the earth.

The United States Air Force presently has three Magnetic Observatories on the ground equipped with Varian magnetometers. These instruments are recording important infor-

mation on the daily variations of the earth's magnetic field and in conjunction with the measurements obtained from the satellite will provide a valuable tool for determining location and strength of the currents responsible for magnetic storms at ground level.

Varian's airborne magnetometer has already undergone one drastic size reduction from the original 150-lb. model to about 22 lb. for the Aerobee Rocket tests where it is performing successfully in providing magnetic data obtained in the upper atmosphere.

The satellite installation of the Varian magnetometer will weigh approximately 3 lb. exclusive of the battery. It will consist of two parts: the "coil and sample," or sensing head, reduced to approximately 2 lb. in weight and half the size of the Aerobee magnetometer; and the minified, transistorized, electronic portion designed to weigh about 1 lb.

Several ground stations will receive the telemetered data from the satellite on magnetic tape and will later use this information to give the magnetic field value at the position of the satellite when it is interrogated. The cycle of measurement from ground signal to recording of the telemetered data will be four seconds of each 90-min. cycle.

New Assault Boat.—A new lightweight, durable assault boat has been developed by the U. S. Army Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va. Carrying its full crew of 15, the pneumatic type craft can attain speeds of 7.1 mph. by use of a 25-hp. outboard motor, or 3.3 mph. by hand-paddling.

Constructed of neoprene-coated nylon, the boat consists of a main flotation tube, an air-mat bottom, a thwart tube connecting the main flotation tube amidship, and a 4-in. diameter spray rail extending around the periphery of the main flotation tube.

The main tube is 18 in. in diameter and is divided into six compartments by hemispherical bulkheads. Division of the tube is a safety feature in that as many as four compartments can be damaged without the boat being knocked out of action.

The boat is equipped with two large and one small inflation-deflation pumps, a repair kit and eleven 5-ft. paddles. A carrying case fabricated of cotton duck is also provided. With equipment, it weighs 255 lb. Inflated size is 17 ft. long, 5 ft, 8 in. wide

The boat is air droppable and can be readily carried by six men. Due to its weight and construction, more than twice as many pneumatic boats can be carried on a 2½-ton truck than the conventional rigid type boats.

Engineering tests have been completed and additional boats have been procured for troop tests.

"Human Barrier" Under Investigation.—Scientists and medical specialists are preparing an experiment under the auspices of the Air Research and Development Command in an effort to crack aviation's "human barrier." It is hoped this can be done by studying the physical and psychological effects on aircrews who are confined on extended "flights" in small, crowded cabins.

A full-scale mock-up of a small (approximately 6 ft. by 7 ft. by 17 ft.) "flight station" will be used to confine five-man Air Force crews for 120 hrs. each in these series of tests.

The tests will be conducted for ARDC by Lockheed Aircraft Corporation at its Marietta, Ga., plant, and monitored by ARDC's Wright Air Development Center, which is conducting a similar experiment at Dayton.

For periods of nearly one week, five men will be confined in an area about the size of a normal automobile. They will "fly" the plane—operating dummy controls and simulating both normal and emergency conditions. They will eat, sleep, work, and relax in this tiny area. Ten crews will be used during the experiment, which is expected to end early in 1958. Psychologists and physicians will examine the men before, during, and after the tests.

While other similar studies have been made, the current investigations are unique in their attempts to measure, in a controlled laboratory situation, the effects of confining crews to a minimal space for an extended period of time.

Solid-State Microwave Amplifier Uses Ferrites.—A new solid-state microwave amplifier using a ferrite material as the active element has been successfully operated at Bell Telephone Laboratories. The ferromagnetic amplifier was predicted on theoretical grounds by Dr. H. Suhl, and the experimental program was carried out by Dr. M. T. Weiss, both of whom are research physicists at Bell Laboratories.

Although still in the laboratory stage, this amplifier is an important addition to the growing family of solid-state devices. It operates at room temperature, and is expected to have a much lower noise level than conventional microwave amplifiers. Thus it has exciting possibilities as an amplifier for very weak microwave signals such as may be encountered in the fields of radio astronomy, microwave relaying and radar.

This amplifier has certain superficial similarities to the solid-state spin oscillator which was proposed by Prof. N. Bloembergen of Harvard University and first made to oscillate by Drs. H. E. D. Scovil, G. Feher and H. Seidel of Bell Laboratories, as announced in January, 1957. Both types must be supplied power from an oscillator operating at a higher frequency than the signal to be amplified, but the principles of operation of the two devices are quite different.

In principle, the present device requires that a ferrite sample be placed in a microwave cavity which is simultaneously resonant at two signal frequencies. Microwave power at a frequency equal to the sum of the two signal frequencies is then pumped into the cavity. A d-c. magnetic field, properly oriented and of sufficient intensity to cause gyromagnetic resonance at this sum frequency, must also be applied. Through nonlinear coupling in the ferrite, amplification or oscillation will be exhibited at either of the lower frequencies, or frequency conversion of a microwave signal can take place between these two frequencies.

In one experiment conducted at Bell Laboratories, the two signal frequencies were each made approximately 4500 mc., to simplify the circuitry. The pumping power of 9000 mc. was fed into the resonant cavity and the signal power taken in and out by means of coaxial cable. With sufficient pumping power, oscillations took place at 4500 mc. When the pumping power was reduced somewhat, amplification was observed at 4500 mc.

In the experimental setup, the d-c. magnetic field (about 2500 gauss) was provided by an electromagnet, but a permanent magnet could be employed if desired. The field is parallel to the plane of the strip-line cavity and oriented at an angle of about 45° with the cavity.

In another experiment, a cavity was designed to be resonant at the two frequencies of 4000 and 4800 mc.,

with the pumping frequency set at 8800 mc. Oscillations and frequency conversion were observed at both the lower frequencies.

The action of the ferromagnetic amplifier can be simulated at low frequencies by a pair of ordinary resonant circuits coupled by a variable inductance or capacitance. Since certain very high frequency atomic phenomena also have this circuit analog, it is possible that the same amplification principle will find applications at frequencies much higher than those presently in use.

It appears that ferromagnetic amplifiers can be designed for operation in practically any portion of the microwave spectrum. Preliminary results indicate that the bandwidth is adequate for many applications. Thus, although the new device is still in the experimental stage in the research laboratory, this discovery may lead the way to an entirely new group of ferrite microwave devices.

Inexpensive Exposure Meter.—General Electric has announced the first American exposure meter designed for the budget-minded, beginners' market. Called the MASCOT II, the direct-reading meter will sell for about $10.00.

Photometric engineers at the company's Instrument Department—manufacturers of precision measurement devices for 75 years—describe the MASCOT II as no bigger than a cigaret lighter, but capable of performance well within ASA accuracy standards.

The budget-priced meter gives instant "one-answer" exposure readings and is extremely simple to use, the photometric ingineers said.

Useful product features of the MASCOT II include a direct-reading scale for both f-stops and the new Exposure Values (EVS), and settings for movie cameras at 16, 24 and 48 frames per second.

"Carbon-Tet" Vapor Detector.—Medical detection of carbontetrachloride poisoning—potentially four times as dangerous as carbon-monoxide—may be greatly improved through tests conducted by a General Electric engineering medical-team. Carbon-tetrachloride is a chemical used in many consumer cleaning products and in various industrial solvents. Poisoning can result simply from breathing carbon-tetrachloride vapors.

Preliminary investigations by the company's Instrument Department engineers and Dr. J. W. Blevins, plant physician, show that the General Electric Halogen leak detector will detect carbon-tetrachloride in the human system more than an hour after exposure to relatively small amounts of "carbon-tet" vapors.

Dr. Blevins said other instruments have been used for this purpose, but "the increased sensitivity and accuracy of the H-1 detector greatly improves medical tests for detecting carbon-tetrachloride poisoning."

General Electric's detector obtains a quantitative measurement of the gas present as well as a qualitative detection. This, according to Dr. Blevins, has made the instrument applicable to medical uses and extremely valuable to the industrial hygienist for monitoring areas where toxic halogen gases may be used.

The supersensitive Halogen leak detector—an instrument used primarily to locate leaks in tightly sealed industrial systems such as refrigerator and air conditioner units—can find a leak so small that only one ounce of gas would leak through the opening in 100 years.

BY-LAWS *

ARTICLE I

NAME AND OBJECT

Section 1. The name of this institution shall be "The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts."

Section 2. The object of The Institute shall be the promotion and encouragement of manufactures and the mechanic and useful arts, by the establishment of lectures on the sciences connected with them, by the formation of cabinets of models, minerals, machines, materials, and products, by exhibitions, and premiums, by a library, and by all such measures as may be judged expedient.

ARTICLE II

MEMBERSHIP AND DUES

Section 1. All persons interested in the object set forth in Article I and in the general activities of The Institute and willing to further them may become members upon election by the Board of Managers or in a manner prescribed by the Board, and upon payment of the prescribed dues.

Section 2. Membership shall consist of the following classes:

1. Annual Membership
2. Life Membership
3. Student Membership
4. Honorary Membership

Section 3. ANNUAL MEMBERSHIP. An annual member, who shall be not less than eighteen years of age, shall be entitled to all membership privileges, including the right to vote and to hold office, free admission to the Museum and Planetarium, the use of the Library, and the right to subscribe to the *Journal* at a reduced price fixed by the Board of Managers. The members of the family of an annual member, residing in his household, shall be entitled to free admission to the Museum and Planetarium. The dues for annual membership shall be $15.00 a year, except that a member residing fifty miles or more from Philadelphia, a member enrolled as an Associate Member when these By-Laws are adopted, and a member who is in the teaching profession on the staff of an elementary school, a high school or its equivalent, or college, shall pay annual dues of $7.50.

Section 4. LIFE MEMBERSHIP. A person may become a member for life, with all the privileges of an annual member, upon the payment of the sum of $300.00 in one year.

Section 5. STUDENT MEMBERSHIP. A student member shall be not less than eight nor more than twenty-five years of age, and shall be registered for full-time day or evening work at an elementary school, high school or its equivalent, or college. He shall receive free admission to the Museum and Planetarium, the right to subscribe to the *Journal* at the same reduced rate as an annual member, and the right to apply for Library privileges after attaining the age of fourteen years, such privileges to conform to such rules as shall be prescribed by the Librarian. He shall have no right to vote or to hold office. The dues for student membership shall be $3.00 a year.

Section 6. HONORARY MEMBERSHIP. An honorary member may be nominated by the Board of Managers at any meeting of the Board, and such nomination shall be presented to The Institute at its next Stated Meeting. An individual so nominated may be elected by the affirmative vote of four-fifths of the voting members present at such meeting. An honorary member shall be entitled to all the privileges of an annual member except those of voting and holding office, but shall not be subject to the payment of dues. He shall also receive the *Journal* free of charge.

Section 7. Dues for membership shall be billed annually on the first day of the month of election. Any member whose dues have not been paid within six months from the date they become due, shall forfeit his membership.

Section 8. The Board of Managers may remit temporarily in whole or in part the dues of any member of The Institute either by action in an individual case or by the establishment of regulations for certain classes.

Section 9. Resignation of membership

* Adopted at the Stated Meeting of The Franklin Institute held May 15, 1957, and effective beginning July 1, 1957.

shall be made to the Board of Managers in writing but need not be accepted until all dues in arrears up until date of resignation shall have been paid. Requests for transfers of membership from one class to another shall be made to the Board of Managers in writing.

ARTICLE III

FRIENDS OF FRANKLIN

Section 1. Any contributor by gift to the annual operating funds of The Institute shall be designated a *Friend of Franklin* for the ensuing year.

Section 2. An individual Friend of Franklin whose gift is in excess of the annual membership fee may be elected to membership in The Institute with his membership fee deducted from such gift and without further payment of such fee.

ARTICLE IV

BOARD OF MANAGERS

Section 1. The Institute shall be governed by a Board of thirty (30) Managers elected by the voting members from among their own number. They shall be classified in respect to the time for which they severally hold office. The Board of Managers shall have final responsibility for and control of the affairs of The Institute and of the Benjamin Franklin Memorial. Through the President the Board shall present a report on the operations and condition of The Institute to the members thereof at a Stated Meeting within six months of the close of each year.

Section 2. Each class of Managers shall consist of no more than ten members, the term of office of each class expiring respectively on the date of the Annual Meeting of The Institute for each of the succeeding three years. At each Annual Meeting of The Institute, ten Managers shall be elected by the members to serve for three years or until their successors shall have been elected.

Section 3. Upon the adoption of these By-Laws, the present Managers shall be continued in office for the terms for which they are presently classified, subject to any reclassification as in this Section 3 provided for. The vacancies created in the Board of Managers, by reason of the increase in its membership as provided in Section 1 of this Article IV, shall be filled by the Board of Managers from among the present officers and such other members of The Institute as may be required, the Board of Managers classifying all such persons assigned to such vacancies as it sees fit to and among the three classes of existing Managers whose respective terms of office expire in the three successive years following the adoption of these By-Laws.

Section 4. A member of the Board of Managers who has not attended five regular meetings of the Board in any calendar year prior to the meeting of the Board in December, shall be reported thereat by the Secretary as having resigned, unless it be unanimously voted by the members of the Board of Managers present at the December meeting that such Manager had been absent for sufficient cause.

Section 5. Nominees for Managers to be elected at the Annual Meeting shall be chosen from the voting members of The Institute. Nominations shall be made in writing and presented to the Stated Meeting of The Institute in the month of December. Each nomination paper must be signed by at least two voting members of The Institute who shall certify that the candidate will serve if elected. After the nominations are closed, the presiding officer shall appoint three voting members, who are neither officers nor nominees, to act as tellers of election. The list of nominees shall be incorporated (with directions for voting) in a ballot to be sent by the Secretary to each member entitled to vote, at least ten days before the date of the Annual Meeting. Each ballot shall be provided with a space for the signature of the member voting and shall be accompanied by a return envelope addressed to the *Tellers of Election.*

Section 6. On the day of the Annual Meeting, at an hour previously designated by their Chairman, the tellers shall meet at The Institute, and shall count all legal votes that have been received by mail or placed in the ballot box before eight o'clock P.M.; and when the count is completed they shall report to the Annual Meeting of The Institute the total number of ballots cast, together with the number of votes received by each candidate. Thereupon the presiding officer shall announce the names of the

candidates who received the plurality of votes to fill the existing vacancies, and shall declare them elected Managers of The Institute for the ensuing years.

Section 7. No election at the Annual Meeting shall be valid unless it results from the vote of at least 48 members, cast as hereinbefore set forth.

Section 8. Any vacancy occurring on the Board of Managers may be filled by the Board by the election of a person to serve until the next Annual Meeting of The Institute or until his successor shall have been elected.

Section 9. The Board shall hold a stated meeting in each month except in July and August at a time to be set by resolution of the Board. The annual organization meeting of the Board for the election of officers, appointment of committees, and such other business as may come before the meeting shall be held on the fourth Wednesday of January or at such other time as shall be determined by resolution of the Board. A special meeting of the Board may be called by the President at his discretion and shall be called by him within ten days after the written request of any five members of the Board. In case of his absence, refusal or inability to act, such special meeting shall be called by the Secretary.

Section 10. A quorum for the transaction of business at all meetings of the Board shall consist of eleven members.

ARTICLE V

EXECUTIVE COMMITTEE

Section 1. The Board of Managers may elect an Executive Committee consisting of five of its members with such authority to act on behalf of the Board as the Board may delegate. Three members shall constitute a quorum at all meetings of the Executive Committee.

ARTICLE VI

OFFICERS

Section 1. The Officers of The Institute shall consist of a President, an Executive Vice President, such other Vice Presidents, if any, as may be determined from time to time by the Board of Managers, a Treasurer, a Secretary, a Director of Laboratories, an Assistant Treasurer, and an Assistant Secretary, all of whom shall be members of The Institute. The Board of Managers may establish such other offices as it deems necessary or advisable.

Section 2. All officers shall be elected by the Board of Managers at its organization meeting, with the President being chosen from the members of the Board, to serve for one year or until their successors shall have been elected. A vacancy occurring in any office may be filled by the Board of Managers at any stated or special meeting of the Board, the officer so elected to fill such vacancy to serve until the next annual organization meeting of the Board or until his successor shall have been elected.

Section 3. The Board of Managers shall determine and fix the compensation, if any, to be paid to each officer.

ARTICLE VII

DUTIES OF OFFICERS

Section 1. The President shall be the executive head of The Institute and shall have general supervision of the affairs of The Institute. He shall preside at all meetings of The Institute and of the Board of Managers. Subject to the approval of the Board, he shall appoint the members of all committees of The Institute and of the Board except the Executive Committee; and he may designate himself a member of such committees as he sees fit.

Section 2. The Executive Vice President, under the supervision of the President, shall have charge of all the affairs of The Institute, and shall be responsible for its educational, scientific and general operational management. Subject to these By-Laws and to such regulations as shall be established by the Board of Managers, he shall determine the duties and approve the compensation of all employees of The Institute other than officers, and shall exercise such additional authority as from time to time may be prescribed by the Board of Managers. He shall prepare the annual operating and capital budgets and submit them to the Board of Managers for approval. He shall submit to the Board of Managers at each stated meeting a report on the current operations of The Institute and its various departments. He shall be *ex officio* a non-voting member of each standing committee of the Board,

and if he so desires may attend meetings of the General Committees of The Institute.

Section 3. In the absence or disability of the President, the Executive Vice President shall exercise his duties. In the absence or disability of the Executive Vice President, the President shall designate another officer or other officers to assume the duties of the Executive Vice President. In the absence or disability of both the President and the Executive Vice President, the duties of such officers shall be assigned to such persons or person as the Board of Managers shall designate.

Section 4. Vice Presidents shall have such duties as may be prescribed by the Board of Managers.

Section 5. The Treasurer shall collect and have custody of all monies payable to The Institute together with all monies receivable from the Investment Committee, depositing them in the name of The Institute in such bank or banks, and disbursing them by checks signed, as the Board of Managers may direct. He shall keep account of these receipts and disbursements, shall have general supervision of the financial books of The Institute, and shall render such financial statements as the President or the Board shall direct. He shall report to the Executive Vice President. In the absence or disability of the Treasurer, the Assistant Treasurer shall perform his duties. The Treasurer, Assistant Treasurer, and all other employees whose duties make it advisable shall, at the Institute's expense, give bond in an amount and with such surety as the Board of Managers shall determine. The duties of the Treasurer and of the Assistant Treasurer may from time to time be modified by resolution of the Board of Managers.

Section 6. The Secretary shall give notice of all meetings of The Institute and of the Board of Managers and shall be responsible for keeping the minutes of the proceedings of such meetings. He shall have custody of the seal, records, and official documents of The Institute and in general shall perform the duties usually pertaining to the office of Secretary. He shall report to the Executive Vice President. In the absence or disability of the Secretary, an Assistant Secretary shall perform his duties.

Section 7. The Director of Laboratories shall have immediate charge of the scientific and business activities of The Franklin Institute Laboratories. He shall report to the Executive Vice President.

ARTICLE VIII

STANDING COMMITTEES OF THE BOARD

Section 1. There shall be the following Standing Committees of the Board of Managers:

1. Nominating Committee
2. Budget Committee
3. Personnel Policy Committee
4. Buildings and Grounds Committee
5. Education Committee
6. Research and Development Committee
7. Bartol Research Foundation Committee
8. Biochemical Research Foundation Committee
9. Educational Radio and Television Committee

The Board may provide for such other Standing Committees as it deems advisable.

Section 2. Each Standing Committee shall consist of not less than three nor more than ten members of the Board as the latter may determine. In addition, the Executive Vice President shall be *ex officio* a non-voting member of each Standing Committee. The members of each Standing Committee shall be appointed and the Chairman thereof designated by the President and confirmed by vote of the Board at the annual organization meeting. The members of any additional Standing Committees authorized by the Board of Managers shall be appointed by the President and shall be confirmed by vote of the Board at its next meeting. A vacancy on any of these Committees may be filled by appointment by the President confirmed by the Board at its next meeting. Members of each Standing Committee shall serve for one year or until their successors shall have been appointed.

Section 3. The Nominating Committee shall recommend names of nominees for Manager to be presented to the members of The Institute for their consideration in accordance with Article IV, Section 5, and the names of nominees for officers to be presented to the organization meeting of the Board, in accordance with Article VI, Section 2. At other times as may be necessary, this Committee shall present nominations to

the Board of Managers to fill vacancies on the Board or among The Institute's officers, as provided in Article IV, 8 and Article VI, Section 2, respectively.

Section 4. The Budget Committee shall review the annual operating and capital budgets of The Institute (not including the budget of the Bartol Research Foundation) as presented by the Executive Vice President and then shall submit them with its recommendations to the Board for action not later than the December meeting.

Section 5. The Personnel Policy Committee shall review at least annually the salary scale of The Institute and shall act on behalf of the Board in approving changes in the salary scale. This Committee shall act on behalf of the Board in approving increases in salaries of employees above a certain amount to be determined by the Board, except compensation of officers fixed in accordance with Article VI, Section 3. It shall report such actions to the Board. The Committee shall review from time to time other matters of personnel compensation and welfare—including pensions, insurance and retirement—and shall make recommendations to the Board for changes in policy in these matters as it sees fit.

Section 6. The Buildings and Grounds Committee shall have general supervision of the buildings and grounds occupied by The Institute. It shall inspect them at least annually and shall report to the Board on their condition, with recommendations, if any.

Section 7. The Education Committee shall have general supervision over the educational activities of The Institute. It shall include a member from each of the following five committees: Museum and Memorial Committee, Library Committee, Meetings Committee, Publications Committee, and the Educational Radio and Television Committee. The Education Committee shall review the educational activities of The Institute. It shall report on them to the Board and shall recommend to the Board such changes in policy as it may deem advisable.

Section 8. The Research and Development Committee shall have general supervision over The Franklin Institute Laboratories and over any other research and development activities of The Institute, except those of the Bartol Research Foundation and of the Biochemical Research Foundation. One of its members shall be a member of the Laboratories Committee provided for in Article X. The Research and Development Committee shall review semi-annually the financial status of the Laboratories and the general scope of their activities. The Committee shall report to the Board on these matters and shall recommend to the Board such changes in policy as it may deem advisable.

Section 9. The Bartol Research Foundation Committee shall have general supervision of the affairs of the Foundation created under the will of the late Henry W. Bartol. The Committee shall be aided by a *Bartol Visiting Committee*, which shall consist of not less than five distinguished physicists or other scientists (not necessarily members of The Institute) appointed by the Bartol Research Foundation Committee. The Chairman of the latter committee shall serve as Chairman of the Bartol Visiting Committee. The Visiting Committee shall inspect the laboratories of the Bartol Research Foundation at least semi-annually and at such times shall review the scientific program with the staff. The Visiting Committee shall make a written report semi-annually to the Bartol Research Foundation Committee, with its recommendations for changes, if any, in the program or facilities of the Foundation. The Bartol Research Foundation Committee in turn shall transmit this report with its own recommendations to the Board of Managers. It shall also review the annual budget of the Foundation as presented by the Executive Vice President and then shall submit it with its recommendations to the Board for action not later than the December meeting.

Section 10. The Biochemical Research Foundation Committee shall see that the obligations of The Institute as Trustee for the Foundation are performed and that the rights of The Institute are preserved. It shall make a review at least annually of the facilities and operations of the Foundation and report on them to the Board. It shall be available to the Advisory Council of the Foundation for collaboration and advice.

Section 11. The Educational Radio and Television Committee shall supervise the performance of the obligations and protect the interests of The Institute with respect to its agreements with the Metropolitan Philadelphia Educational Radio and Television

Corporation and with any other body, corporate or otherwise, operating educational radio and television facilities in which The Institute, as Trustee, owner, or otherwise, has any proprietary or contractual interest. It shall keep itself informed regarding the operations of the facilities and shall make such inspections of them as it may deem advisable (at lease annually) and shall report to the Board concerning these matters, with such recommendations as it may deem advisable.

Section 12. Each Standing Committee shall meet at such times and places as the Committee or its Chairman shall decide. A quorum for the transaction of business at all meetings of the Standing Committees shall be a majority of the voting members.

ARTICLE IX

GENERAL COMMITTEES OF THE INSTITUTE

Section 1. There shall be the following General Committees of The Institute:

1. Committee on Science and the Arts
2. Membership Committee
3. Investment Committee
4. Endowment Committee

Section 2. The size of each Committee shall be determined by the Board of Managers. The members of each General Committee of The Institute shall be selected from the voting members of The Institute. The members of the General Committees shall be appointed by the President and confirmed by the Board at its annual organization meeting, provided that the appointments to the Committee on Science and the Arts shall be made from members of The Institute nominated in writing by that Committee. The Chairman of the Committee on Science and the Arts shall be designated by that Committee. The Chairman of each of the other General Committees shall be a member of the Board of Managers designated by the President. A vacancy occurring on any General Committee may be filled by the President at any stated meeting of the Board subject to confirmation by the Board. Members of General Committees shall serve for one year or until their successors shall have been appointed, except that the members of the Committee on Science and the Arts shall serve for three years as described in Section 4 of this Article.

Section 3. Each General Committee shall meet at such times and places as the Committee or its Chairman shall decide. A quorum for the transaction of business at all meetings of the Committee on Science and the Arts shall be fifteen; for the other General Committees, a majority of the members shall constitute a quorum.

Section 4. The Committee on Science and the Arts shall consist of not less than sixty or more than seventy-five members divided into three classes approximately equal in number, the term of office of each class to expire respectively on the date of the annual organization meeting of the Board of Managers in successive years. The Committee shall investigate current discoveries, inventions and other achievements in the sciences and their applications in the mechanical and industrial arts with a view to recommending to the Board of Managers for its action such awards as may lie within the power of The Institute to bestow, exercising independent judgment in all such investigations. The Committee shall keep a record of all its proceedings which shall be reported to the Board of Managers when the latter so desires. In all matters of an administrative nature the Committee shall collaborate with the Executive Vice President. The Chairman of this Committee, if not a member of the Board of Managers, is expected to attend stated meetings of the Board.

Section 5. The Membership Committee shall review monthly the applications for membership received during the preceding calendar month, and, when authorized by the Board, shall elect to membership such applicants as to it may seem proper. It shall render a report of its actions to the Board at the next meeting of the Board. The Committee shall collaborate with the Executive Vice President in expanding the membership of The Institute.

Section 6. The Investment Committee, subject to the supervision of the Board of Managers, shall have the custody and control of all the securities and investments of The Institute with full power and authority to buy and sell, and to invest and reinvest the same, including the power to satisfy mortgages and extinguish ground rents, and to direct the placing of all such insurances on real estate held for investment as

it may deem necessary, and to make such improvements, repairs and alterations to such real estate as it may deem necessary. It shall have power to: authorize the proper officers of The Institute to execute the necessary papers to effect all purchases, sales and assignments of property other than real estate; transfer registered securities; subscribe to bond-holders' agreements to plans of reorganization involving any securities held by The Institute or in which it has an interest; and do all such acts as are necessary in pursuance of the foregoing powers. The Committee may, with the approval of the Board of Managers, appoint a Trust Company in the City of Philadelphia to act as Fiscal Agent under the direction of the Committee. The Investment Committee shall keep a record of all its acts and proceedings, which shall be communicated to the Board of Managers.

Section 7. The Endowment Committee shall be charged with all fund-raising activities of The Institute and shall collaborate with the Executive Vice President in promoting such activities. It shall report to the Board of Managers from time to time (at least annually) on these activities.

ARTICLE X

ADVISORY COMMITTEES OF THE INSTITUTE

Section 1. There shall be the following Advisory Committees of The Institute:

1. Museum and Memorial Committee
2. Library Committee
3. Meetings Committee
4. Publications Committee
5. Laboratories Committee

The Board of Managers may provide for such other Advisory Committees as it deems advisable.

Section 2. The members of the Advisory Committees shall be selected from the voting members of The Institute, provided that at least one member of each Committee shall be a member of the Education Committee or the Research and Development Committee as specified in Sections 7 and 8 of Article VIII. The members of each Advisory Committee shall be appointed and the Chairman thereof designated by the President and confirmed by vote of the Board at its annual organization meeting. The number of members on Advisory Committees need not be fixed, and additional members may be appointed at any meeting of the Board, for a term expiring at the date of the following organization meeting of the Board.

Section 3. The Advisory Committees shall be of counsel and advice to the President and to the staff members in charge of the several operations indicated by the names of the committees. Each Advisory Committee shall meet at such times and places as the Committee or its Chairman may decide; and in addition it shall meet at the request of the President to consider any particular matter on which he would like advice.

ARTICLE XI

MEETINGS OF THE INSTITUTE

Section 1. Stated Meetings of The Institute shall be held on the third Wednesday of each month, except June, July, August and September, at such hour and place as shall be decided by the Board of Managers.

Section 2. The Annual Meeting of The Institute shall be held on the third Wednesday of January each year, at such hour and place as the Board of Managers may decide.

Section 3. A special meeting shall be called by the President upon the order of the Board of Managers within ten days thereof or upon the written application of twenty-four voting members of The Institute. In case of his absence, refusal, or inability to act, such special meeting shall be called by an officer designated by the Board of Managers.

Section 4. Except as otherwise provided in these By-Laws, twenty-four members shall constitute a quorum at all meetings of The Institute.

ARTICLE XII

AMENDMENTS

Section 1. These By-Laws may be altered or amended at any Stated Meeting of the members of The Institute by majority vote of the voting members present, provided notice in writing of the proposed alteration or amendment signed by at least two voting members shall have been given to the Board of Managers at least two months prior to the said meeting, and shall have been communicated by the Board to the membership at least one month prior to the said meeting.